Comparison of grand unified theories with electroweak and strong coupling constants measured at LEP

Ugo Amaldi [1]
CERN, CH-1211 Geneva 23, Switzerland

Wim de Boer [2] and Hermann Fürstenau [3]
Institut für Experimentelle Kernphysik, Universität Karlsruhe, Postfach 6980, W-7500 Karlsruhe 1, FRG

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Using the renormalization group equations one can evolve the electroweak and strong coupling constants, as measured at LEP, to higher energies in order to test the ideas of grand unified theories, which predict that the three coupling constants become equal at a single unification point. With data from the DELPHI Collaboration we find that in the minimal *non-supersymmetric* standard model with one Higgs doublet a single unification point is excluded by more than 7 standard deviations. In contrast, the minimal *supersymmetric* standard model leads to good agreement with a single unification scale of $10^{16.0\pm0.3}$ GeV. Such a large scale is compatible with the present lower limits on the proton lifetime. The best fit is obtained for a SUSY scale around 1000 GeV and limits are derived as function of the strong coupling constant. The unification point is sensitive to the number of Higgs doublets and only the minimal SUSY model with two Higgs doublets is compatible with GUT unification, if one takes the present limits on the proton lifetime into account.

1. Introduction

Since August 1989 LEP has been producing electron–positron collisions at various center of mass energies centered around the peak for resonance production of Z^0 gauge bosons ($M_Z \approx 91.17$ GeV). The four experiments (ALEPH, DELPHI, L3, and OPAL) have been collecting between 150 000 and 200 000 events each and analyses on the most varied subjects have been performed. The two most publicized results are the very precise measurements of the number of standard light neutrinos and the absence of a standard neutral Higgs boson in a mass interval between 0 and about $M_Z/2$. In this note we elaborate on a third result: the precision of the LEP data allows to extrapolate the three coupling constants of the standard model (SM) to high energies with small errors, thus allowing to perform consistency checks of grand unified theories (GUT). These theories reduce the number of independent coupling constants by embedding the $SU(3)_C \otimes SU(2)_L \otimes U(1)$ group of the standard model (SM) into a group G, in which there is only one underlying gauge coupling constant at the unification scale. If at this scale the group G is broken into the three independent gauge groups, the evolution of the corresponding coupling constants depends only on the particle content of the $SU(3)_C \otimes SU(2)_L \otimes U(1)$ model, but not on the structure of the group G. By evolving the coupling constants, as measured at low energies, towards high energies one can check which particle content is consistent with unification.

Our starting point is the analysis performed in 1987 in ref. [1]. One of the results is summarized in fig. 1 (fig. 11 from ref. [1]), where the inverses of the standard model couplings are plotted versus the energy. Clearly, around 10^{15} GeV all couplings become of the same order of magnitude, but they do not meet exactly in a single unification point. The deviation from perfect crossing, as expected in the simplest

[1] Bitnet address: AMALDI@CERNVM.
[2] Bitnet address: DEBOERW@CERNVM.
[3] Bitnet address: FURSTENA@CERNVM.

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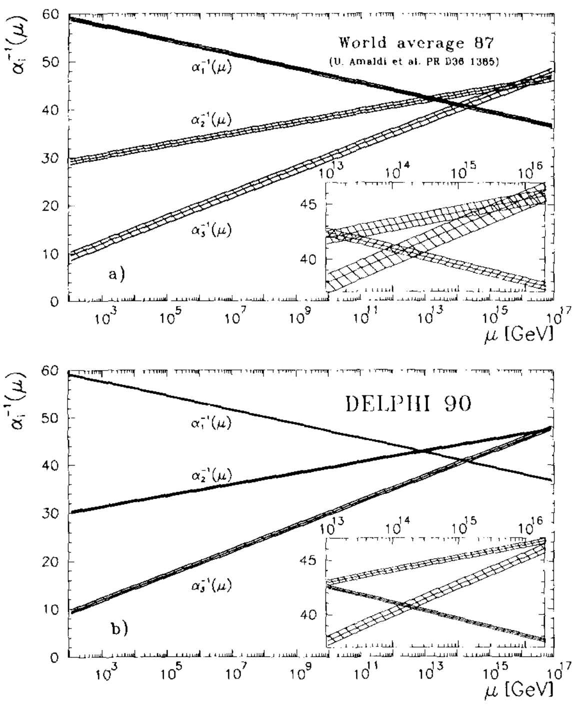

Fig. 1. (a) First order evolution of the three coupling constants in the minimal standard model (world average values in 1987 from ref. [1]). The small figure is a blow-up of the crossing area). (b) As above but using M_Z and $\alpha_s(M_Z)$ from DELPHI data. The three coupling constants disagree with a single unification point by more than 7 standard deviations.

unified theories, was only two standard deviations in 1987.

In this paper we extend that analysis with recent more precise LEP data. We do this along lines similar to the ones recently adopted by Ellis et al. [2] and Langacker [3]. We use published data from the DELPHI Collaboration, of which we are members, but similar results could be derived from the data of other LEP experiments.

The paper has been organized as follows: In sections 2 and 3 the coupling constants are defined and their new determinations are described. In section 4 the evolutions of the coupling constants to high energies in the minimal standard model (SM) and in the minimal supersymmetric standard model (SUSY) are compared. A summary is given in section 5.

2. Definition of the coupling constants

In the unified $SU(2)_L \otimes U(1)$ theory, the following well known relations hold between the coupling constants and the gauge boson masses:

$$e=\sqrt{4\pi\alpha}=g\sin\theta_w=g'\cos\theta_w\,, \tag{1}$$

$$M_W=\tfrac{1}{2}vg'\,, \tag{2}$$

$$M_Z=\tfrac{1}{2}v\sqrt{g'^2+g^2}\,, \tag{3}$$

from which it follows that

$$\sin^2\theta_w=\frac{e^2}{g^2}=\frac{g'^2}{g'^2+g^2}=1-\frac{M_W^2}{M_Z^2}\,. \tag{4}$$

Here g and g' are the coupling constants of the groups $SU(2)_L$ and $U(1)$, respectively, α is the fine structure constant, θ_w is the electroweak mixing angle and v is the vacuum expectation value of the Higgs field. If the model contains Higgs representations other than doublets, the theory has an additional degree of freedom, usually parametrized by the ρ-parameter.

In the SM based on the group $SU(3)_C\otimes SU(2)_L\otimes U(1)$ the usual definitions of the coupling constants are

$$\alpha_1=\tfrac{5}{3}g'^2/4\pi=5\alpha/3\cos^2\theta_{\overline{MS}}\,, \tag{5}$$

$$\alpha_2=g^2/4\pi=\alpha/\sin^2\theta_{\overline{MS}}\,, \tag{6}$$

$$\alpha_3=g_s^2/4\pi\,, \tag{7}$$

where g_s is the $SU(3)_C$ coupling constant. The factor $\frac{5}{3}$ in the definition of α_1 has been included for the proper normalization at the unification point [4].

The coupling constants, if defined as effective values including loop corrections in the gauge boson propagators, become energy dependent ("running"). A running coupling constant requires the specification of a renormalization scheme (RS). We will use the usual modified minimal subtraction scheme ($\overline{MS}$) [5]. The energy dependence is completely determined by the particle content and their couplings inside the loop diagrams of the gauge bosons, as expressed by the renormalization group equations. The first order renormalization group equations are

$$\mu\frac{\partial}{\partial\mu}\alpha_i(\mu)=\frac{1}{2\pi}b_i\alpha_i^2(\mu)+\ldots,\quad i=1,2,3\,, \tag{8}$$

where μ is the energy at which the couplings are evaluated. For the SM the coefficients are [6]

$$b_i=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}0\\-\frac{22}{3}\\-11\end{pmatrix}+N_{\rm Fam}\begin{pmatrix}\frac{4}{3}\\\frac{4}{3}\\\frac{4}{3}\end{pmatrix}+N_{\rm Higgs}\begin{pmatrix}\frac{1}{10}\\\frac{1}{6}\\0\end{pmatrix},\qquad(9)$$

while for the supersymmetric extension [7] of the SM (to be called SUSY in the following) they have been calculated to be [6]

$$b_i=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}0\\-6\\-9\end{pmatrix}+N_{\rm Fam}\begin{pmatrix}2\\2\\2\end{pmatrix}+N_{\rm Higgs}\begin{pmatrix}\frac{3}{10}\\\frac{1}{2}\\0\end{pmatrix},\qquad(10)$$

where $N_{\rm Fam}$ is the number of families of matter fields and $N_{\rm Higgs}$ is the number of Higgs doublets. In the minimal SM and in the minimal SUSY model $N_{\rm Fam}=3$ and $N_{\rm Higgs}=1$ and 2, respectively. Note that in the supersymmetric model the dominating first order coefficients lead to a much weaker running of α_3 than predicted by the standard model, while the running of α_2 has the opposite sign and α_1 runs somewhat faster.

In second order the renormalization group equations can be written as

$$\mu\frac{\partial}{\partial\mu}\alpha_i(\mu)=\frac{2}{4\pi}\left(b_i+\frac{b_{ij}}{4\pi}\alpha_j(\mu)+\frac{b_{ik}}{4\pi}\alpha_k(\mu)\right)\alpha_i^2(\mu)+\frac{2b_{ii}}{(4\pi)^2}\alpha_i^3(\mu),\qquad(11)$$

where $i, j, k=1, 2, 3$ and $i\neq j\neq k$. The b_i have been given above and the b_{ij} for the standard $SU(3)_C\otimes SU(2)_L\otimes U(1)$ are [6]

$$b_{ij}=\begin{pmatrix}0&0&0\\0&-\frac{136}{3}&0\\0&0&-102\end{pmatrix}+N_{\rm Fam}\begin{pmatrix}\frac{19}{15}&\frac{3}{5}&\frac{44}{15}\\\frac{1}{5}&\frac{49}{3}&4\\\frac{11}{30}&\frac{3}{2}&\frac{76}{3}\end{pmatrix}+N_{\rm Higgs}\begin{pmatrix}\frac{9}{50}&\frac{9}{10}&0\\\frac{3}{10}&\frac{13}{6}&0\\0&0&0\end{pmatrix}.\qquad(12)$$

For the SUSY model they become [6]

$$b_{ij}=\begin{pmatrix}0&0&0\\0&-24&0\\0&0&-54\end{pmatrix}+N_{\rm Fam}\begin{pmatrix}\frac{38}{15}&\frac{6}{5}&\frac{88}{15}\\\frac{2}{5}&14&8\\\frac{11}{15}&3&\frac{68}{3}\end{pmatrix}+N_{\rm Higgs}\begin{pmatrix}\frac{9}{50}&\frac{9}{10}&0\\\frac{3}{10}&\frac{7}{2}&0\\0&0&0\end{pmatrix}.\qquad(13)$$

Note that the $\alpha_i^2(\mu)$ coefficients change when the second order contributions are taken into account and the running of each α_i depends on the values of the other two coupling constants. However, the second order effects are small, because the b_{ij} are multiplied by $\alpha_j/4\pi\leqslant0.01$. Higher orders are presumably even smaller.

Eqs. (11) can be solved by integration to obtain $\alpha_i'(\mu')$ for a given $\alpha_i(\mu)$:

$$\alpha_i'(\mu')=\left[\beta_0\ln\frac{\mu'}{\mu}+\frac{1}{\alpha_i(\mu)}+\frac{\beta_1}{\beta_0}\ln\left(\frac{1/\alpha_i'(\mu')+\beta_1/\beta_0}{1/\alpha_i(\mu)+\beta_1/\beta_0}\right)\right]^{-1},\qquad(14)$$

with

$$\beta_0=\frac{-1}{2\pi}\left(b_i+\frac{b_{ij}}{4\pi}\alpha_j(\mu)+\frac{b_{ik}}{4\pi}\alpha_k(\mu)\right),\qquad(15)$$

$$\beta_1=\frac{-2b_{ii}}{(4\pi)^2}.\qquad(16)$$

This exact solution to the second order renormalization group equation can be used to calculate the coupling constants at an arbitrary energy, if they have been measured at a given energy, i.e. it can be used to calculate $\alpha_i(\mu')$ from a given $\alpha_i(\mu)$. Eq. (14) is most easily solved numerically by iteration.

3. Experimental values for the coupling constants at the Z^0 mass

This section discusses the three input parameters $\sin^2\theta_{\overline{\rm MS}}(M_Z)$, $\alpha(M_Z)$ and $\alpha_s(M_Z)$, which are needed to specify the coupling constants defined in section 2. Numerical values are obtained by combining published data from the DELPHI Collaboration with top mass limits from neutrino scattering and $p\bar{p}$ colliders.

3.1. Determination of $\sin^2\theta_{\overline{MS}}$

The value of $\sin^2\theta_{\overline{MS}}$ can be determined either from direct measurements of the coupling constants at the Z^0 resonance or from the measured value of M_Z alone, using the Sirlin relation [8]

$$\frac{G_F(1-\Delta r)M_Z^2}{8\sqrt{2}\pi\alpha} = \frac{1}{16\sin^2\theta_w\cos^2\theta_w} . \quad (17)$$

We will use the measured value of M_Z [9]

$$M_Z = 91.176 \pm 0.023 , \quad (18)$$

but in this case one has to know the electroweak correction Δr to the muon decay constant G_F, which depends on the top and Higgs masses. Limits on the top mass can be obtained from the average value of $\sin^2\theta_w = 1 - M_W^2/M_Z^2 = 0.2290 \pm 0.0035$, obtained in neutrino scattering [10–12] and $p\bar{p}$ collisions [13, 14]. From this value and eq. (17) one can obtain Δr which in turn can be used to derive limits on M_{top}.

Using a recent calculation of Δr including the QCD and M_{top}^4 corrections [15], we find for $M_{Higgs} = 45(1000)$ GeV a value of $M_{top} = 116 \pm 38 (144 \pm 37)$ GeV. We did not use lower values of the Higgs mass, since 45 GeV is already below the most recent limits from the LEP data [16]. The errors include the uncertainty from the Z^0 mass and the vacuum polarization. The latter error originates from the uncertainty in the hadronic contribution to the QED vacuum polarization as determined from the measured cross section of $e^+e^- \to$ hadrons [17]. In computing the average value of $\sin^2\theta_w$ from neutrino scattering experiments, the above range for the top mass was taken into account.

With the value of M_Z (eq. (18)) and the limits on M_{top} one can calculate the electroweak mixing angle in the $\overline{MS}$ scheme to be [18] $\sin^2\theta_{\overline{MS}} = 0.2340 \pm 0.0014$ for $M_{Higgs} = 1000$ GeV and 0.2331 ± 0.0014 for $M_{Higgs} = 45$ GeV. Averaging the extreme values (0.2317 and 0.2354), one obtains

$$\sin^2\theta_{\overline{MS}} = 0.2336 \pm 0.0018 . \quad (19)$$

3.2. Determination of α

To define the electroweak coupling constants at a scale M_Z we use for the fine structure constant the following parametrization [19]:

$$\alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha} = \frac{1}{128.8} \quad (20)$$

with

$$\Delta\alpha = 0.0602 + \frac{40}{9}\frac{\alpha}{\pi}\log\frac{M_Z}{92\ \text{GeV}} \pm 0.0009 , \quad (21)$$

and $M_Z = 91.176 \pm 0.023$ GeV from eq. (18).

With the value of $\sin^2\theta_{\overline{MS}}$ given above one obtains

$$\alpha_1(M_Z) = 0.016887 \pm 0.000040 , \quad (22)$$

$$\alpha_2(M_Z) = 0.03322 \pm 0.00025 . \quad (23)$$

These values have been obtained with radiative corrections calculated in the standard model with three families and one Higgs doublet. However, given the errors, the contributions of additional particles, which will be considered later, are small [20].

3.3. Determination of α_s.

The present analysis uses the two values of α_s from ref. [21], which are based on the measurements of the differential jet rates and of the asymmetry of the energy–energy correlation, respectively:

$$\alpha_s(M_Z) = 0.114 \pm 0.003[\text{stat.}] \pm 0.004[\text{syst.}] \pm 0.012[\text{theor.}] \quad (24)$$

and

$$\alpha_s(M_Z) = 0.106 \pm 0.003[\text{stat.}] \pm 0.003[\text{syst.}] {}^{+0.003}_{-0.000}[\text{theor.}] . \quad (25)$$

The theoretical errors are due to the renormalization scale dependence [22]. After symmetrizing the theoretical error in the last result, we obtain for the weighted average and its estimated 68% CL error

$$\alpha_3(M_Z) = \alpha_s(M_Z) = 0.108 \pm 0.005 , \quad (26)$$

which corresponds to $\Lambda^{(5)}_{\overline{MS}} = 122^{+47}_{-36}$ MeV and $\Lambda^{(6)}_{\overline{MS}} = 49^{+22}_{-16}$ MeV. The relation between $\Lambda_{\overline{MS}}$ for five and six flavours has been given in ref. [23]. To extrapolate the strong coupling constant to energies larger than the top mass one has to increase the number of active flavours to six. Note that changing the number of flavours does not change abruptly the value of the coupling constant, but only its energy dependence.

This value of α_s agrees with the recent α_s determination from deep inelastic lepton–nucleon scattering and single γ production ($\alpha_s(M_Z)=0.109^{+0.004}_{-0.005}$) [24].

It should be emphasized that many of our conclusions do not depend on the choice of the $\alpha_s(M_Z)$ error. In the relevant part we will repeat the analysis for a range of α_s values.

4. Comparison of the standard model couplings at M_Z with grand unified theories

The simplest grand unified theory is the minimal $SU(3)_C \otimes SU(2)_L \otimes U(1)$ model with three families of matter and one Higgs doublet. The coupling constants should evolve smoothly until they become identical at the unification scale. Here we make the simplifying assumption that at the unification point the couplings cross without changing slopes. We checked that the effect of this simplification for the threshold region is not large compared with our experimental errors [25].

The evolutions of the three coupling constants with the new data are shown in fig. 1b using the minimal SM with three families and one Higgs doublet. Compared to the results of 1987 (fig. 1a), the errors, indicated by the width of the lines, are considerably smaller.

It is clear that a single unification point cannot be obtained within the present errors: the α_3 coupling constant misses the crossing point of the other two by more than 7 standard deviations. Only with $\alpha_s(M_Z)=0.07$ one can force $1/\alpha_3$ to pass through the crossing point of the other two or, alternatively, if one leaves $\alpha_s(M_Z)$ at 0.108, one has to lower $\sin^2\theta_{\overline{MS}}$ to 0.21 in order to get a single unification point. These values are in disagreement with the experimental values quoted in the previous section.

Note that the unification scale is of the order of 10^{15} GeV or less. The proton lifetime, which is proportional to the fourth power of this scale, is then expected to be of the order of 10^{31} yr. Present lower limits are considerably higher: $\tau_{proton} \geqslant 5.5\times10^{32}$ yr [26] for the decay mode $p\to e^+\pi^0$ which is expected to dominate.

Therefore, both the non-observation of proton decay and the non-unification of the coupling constants independently rule out any minimal GUT, which breaks to the SM below the unification point. In the framework of GUT this non-unification implies new physics.

Such new physics could come from supersymmetry [27]. This model introduces a symmetry between bosons and fermions: all fermions (bosons) have a boson (fermion) partner. One of its motivations is the fact that quadratic and Yukawa-coupling divergences in the loop corrections automatically cancel, if the supersymmetric partners have the same couplings and masses as the known particles [28].

In SUSY GUT the evolution equations are modified. We do not know at which scale the modifications start to be effective. Therefore we fitted both the unification scale M_{GUT} and the SUSY breaking scale M_{SUSY}, which is defined as the transition point where the slopes change from the values of eqs. (9) and (12) to the values of eqs. (10) and (13). The fit was done by minimizing the χ^2 for a single unification point, i.e. we minimized

$$\chi^2=\sum_{i=1}^{3}\frac{[\alpha_i(\mu)-\alpha_{GUT}(\mu)]^2}{\sigma_i^2}\,, \qquad (27)$$

σ_i is the error on α_i and $\alpha_{GUT}=\alpha_{GUT}(M_{GUT})$ is the coupling at the unification point. In the minimization the mass of the lightest Higgs doublet was chosen to be equal to M_Z and the mass of the heavier doublet was taken to be equal to M_{SUSY}. These choices have practically no influence on our conclusions as long as the Higgs masses are less than a few times M_{SUSY}.

The second order renormalization group equations give the results shown in fig. 2a. It is apparent from the figure that the three coupling constants meet in a single point around 10^{16} GeV. The resulting χ^2 distributions for M_{SUSY} and M_{GUT} are shown in figs. 2b and 2c. The minimum χ^2 is obtained for

$$M_{SUSY}\approx 1000 \text{ GeV}\,.$$

We have done the extrapolation in first order too, i.e. using eqs. (9) and (10). The difference is within the experimental errors, so the second order contributions are negligible with the present experimental errors.

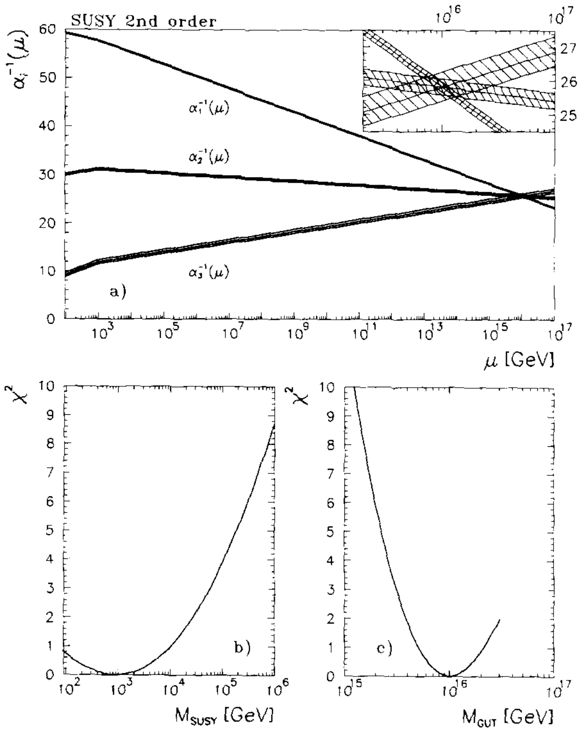

Fig. 2. (a) Second order evolution of the three coupling constants in the minimal SUSY model. M_{SUSY} has been fitted by requiring crossing of the couplings in a single point. The two lower plots show the χ^2 distribution for the SUSY scale M_{SUSY} (b) and for the unification scale M_{GUT} (c) taking into account their correlation.

The widths of the χ^2 distributions are dominated by the error of $\alpha_3(M_Z)$. We have repeated the fits for different values of $\alpha_3(M_Z)$ and the results are shown in figs. 3a and 3b. One observes that M_{SUSY} is a steep function of α_s: for $\alpha_s(M_Z)$ between 0.10 and 0.12, M_{SUSY} varies between 30 TeV and 10 GeV. The 68% CL range of α_3 values, obtained by averaging DELPHI results (see eq. (26)), is also indicated.

Until now the assumption was made that the slopes change from SM values to SUSY values exactly at M_{SUSY}. This abrupt change is unphysical, not only because the particles are virtual, but also because different SUSY particles are likely to have different masses. To model the actual behavior we have smeared this change over 1–3 orders of magnitude symmetrically around M_{SUSY} by taking the average of the SM and SUSY slopes in this interval. This smearing lowers the fitted value of M_{SUSY} and has little influence on M_{GUT}, as shown by the dashed and dotted lines in figs. 3a and 3b.

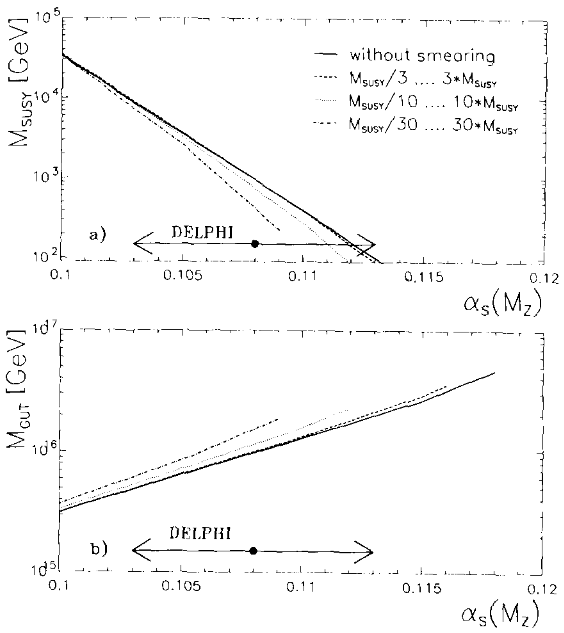

Fig. 3. The M_{SUSY} (a) and M_{GUT} (b) energy scales are shown as function of $\alpha_3(M_Z)$. The uncertainties in M_{GUT} and M_{SUSY} from the errors in $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ are small. The full line assumes that all SUSY particles have the mass of the SUSY scale. The dashed, dotted and dash-dotted lines indicate the results if the SUSY particle spectrum is smeared over the range indicated in the figure.

The values of M_{GUT} and M_{SUSY} are correlated. By taking this correlation into account, one finds

$$M_{SUSY} = 10^{3.0\pm1.0}\ \text{GeV}\,, \tag{28}$$

$$M_{GUT} = 10^{16.0\pm0.3}\ \text{GeV}\,, \tag{29}$$

$$\alpha_{GUT}^{-1} = 25.7\pm1.7\,. \tag{30}$$

Because of the threshold behaviour, the mass of the heavy gauge bosons (M_X) is typically 0.3 M_{GUT} [25]. If the proton decay is dominated by X-boson exchange, the proton lifetime for $M_X = 3\times10^{15}$ GeV can be estimated as

$$\tau_{proton} \approx \frac{1}{\alpha_{GUT}^2}\frac{M_X^4}{M_p^5} = 10^{33.2\pm1.2}\ \text{yr}\,, \tag{31}$$

where M_p is the proton mass. However, the estimate of eq. (31) is not unique because in many SUSY models faster decay processes can contribute [29].

Such a proton lifetime does not contradict the present experimental limits of $\tau_{\rm proton} \approx 10^{32}$ yr [26].

Note that the fitted vale of $M_{\rm SUSY}$ is influenced neither by the number of families nor by their masses, since in first order the slopes of all three coupling constants are changed by equal amounts for each additional family (see eqs. (9) and (10)), so that if there is unification for a certain $M_{\rm SUSY}$, additional families with arbitrary masses do not modify it. This is true in first order and at present the second order effects are small compared with the experimental errors. Only the gauge boson sector or the Higgs sector influence the fitted value of $M_{\rm SUSY}$, once unification is required. Therefore, $M_{\rm SUSY}$ is sensitive to the masses of the gauginos, Higgs bosons and higgsinos.

The errors in eqs. (28)–(31) originate mainly from the error of $\alpha_2(M_Z)$ in the weighted average of eq. (26) and demonstrate the sensitivity of the various quantities to the 5% uncertainty in the knowledge of the strong coupling constant. The three-jet cross section contributes less than the asymmetry in the energy–energy correlation to the weighted average, since its error is dominated by the large second order α_s corrections, which cause a strong renormalization scale dependence (see eq. (24)). We think that every effort should be made to compute $O(\alpha_s^3)$ corrections in order to reduce these theoretical uncertainties and be able to determine $\alpha_s(M_Z)$ with the experimentally well measured jet rates. Of course, the ideal situation would be to have a Monte Carlo with an exact third order QCD matrix element, which could be used to obtain α_s from many different quantities.

Up to now we have compared the data with the minimal SM and SUSY models with one or two Higgs doublets. In the framework of GUT, we have also tested the sensitivity to the number of Higgs doublets, since more doublets could be present in non-minimal models. Additional families do not influence the unification condition, since, as discussed above, the slopes of all three coupling constants are changed by equal amounts and only the gauge boson sector or the Higgs sector influence the unification scale, as pointed out in ref. [30].

Two examples of the sensitivity to the Higgs sector are shown in fig. 4 for:

– the SM with six Higgs doublets at M_Z;

– the SUSY model with one Higgs doublet at M_Z and three additional Higgs doublets at 1 TeV.

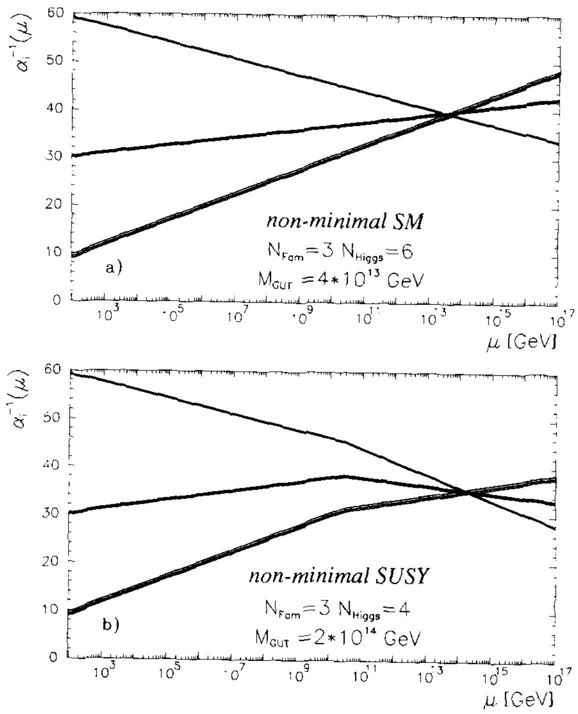

Fig. 4. (a) Best fit to a single unification point within the non-minimal standard model with three families and six Higgs doublets with $M_{\rm Higgs}=M_Z$. (b) Best fit to a single unification point within the non-minimal SUSY model with three families, one Higgs doublet at M_Z and three additional Higgs doublets at 1 TeV.

In both cases the best fits give unification, but the unification point is below 2×10^{14} GeV which, according to eq. (31), is incompatible with the present limits on the proton lifetime [26]. Further fits show that in the non-supersymmetric SM no unification is possible with less than six Higgs doublets. With six or eight doublets the unification scale is too small even when the limitation $M_{\rm Higgs}=M_Z$ is lifted. In the non-minimal SUSY case the unification scale remains low even if the masses of three (or five) additional Higgs doublets are moved from M_Z up to $M_{\rm SUSY}\simeq 10^{10}$ GeV.

In summary, in the GUT framework the minimal SUSY model with two Higgs doublets gives the largest unification scale, and is the only one which may be compatible with the present limits on the proton lifetime. Extra Higgs scalars with masses around $M_{\rm GUT}$ cannot be excluded, since their influence on the extrapolation of the coupling constants becomes small. Higgs singlets cannot be excluded either, since in first order they do not contribute to the running of the coupling constants.

5. Summary

The combination of precise data on the electroweak and strong coupling constants measured at LEP with the limits on the proton lifetime allows stringent consistency checks of unified models.

It was shown that the evolution of the coupling constants within the minimal standard model with one Higgs doublet does not lead to grand unification, but if one adds five additional Higgs doublets, unification can be obtained at a scale below 2×10^{14} GeV. However, such a low scale is excluded by the limits on the proton lifetime.

On the contrary, the minimal supersymmetric extension of the standard model leads to unification at a scale of $10^{16.0\pm0.3}$ GeV. Such a large unification scale is compatible with the present limits on the proton lifetime of about 10^{32} yr. Note that the Planck mass (10^{19} GeV) is well above the unification scale of 10^{16} GeV, so presumably quantum gravity does not influence our results.

It was also shown that non-minimal SUSY models with four or more Higgs doublets – having masses around or below the SUSY scale – yield unification. However, once more, the unification scales are below the limits allowed by the proton decay experiments. Therefore only the minimal SUSY model gives a unification scale which may be compatible with the proton lifetime limit.

The best fit to the allowed minimal SUSY model, shown in fig. 2, is obtained for a SUSY scale around 1000 GeV or, more precisely, $M_{\rm SUSY}=10^{3.0\pm1.0}$ GeV, where the error originates mainly from the uncertainty in the strong coupling constant. If this minimal supersymmetric GUT describes Nature, SUSY particles, which are expected to have masses of the order of $M_{\rm SUSY}$, could be within reach of the present or next generation of accelerators.

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