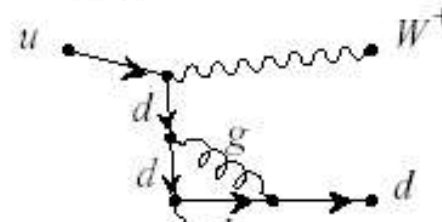
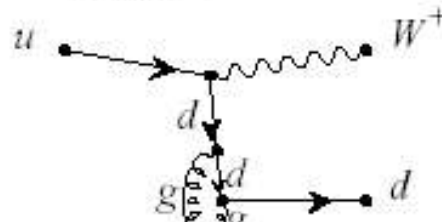
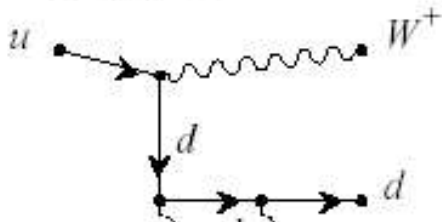
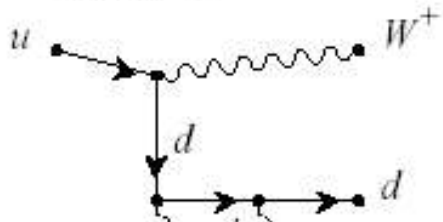


Graph 5

Graph 6

Graph 7

Graph 8



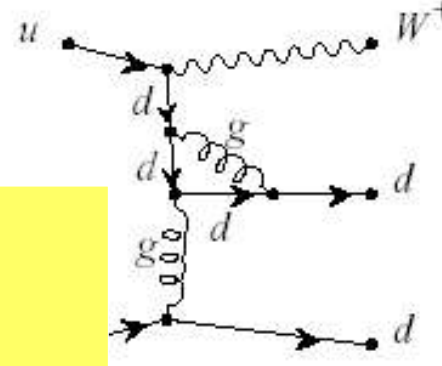
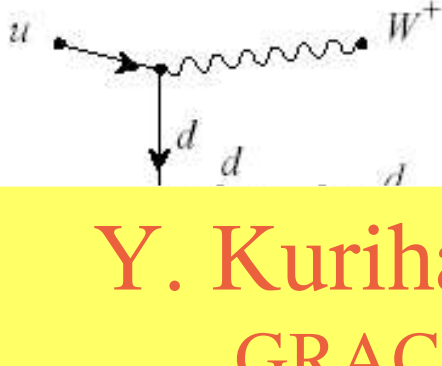
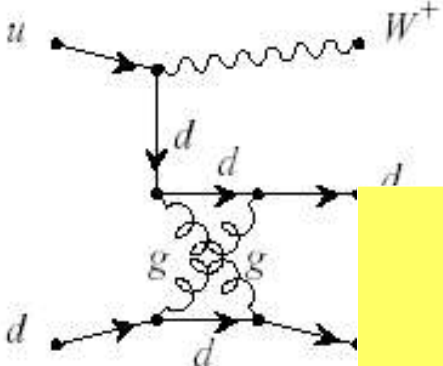
# NLO event generator for LHC

Graph 9

Graph 10

Graph 11

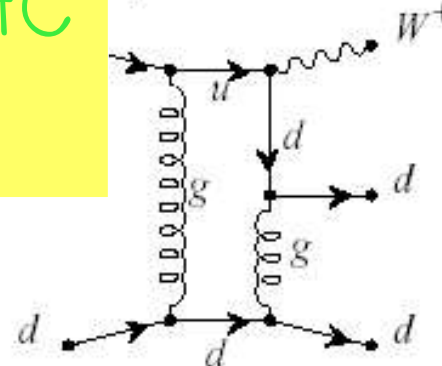
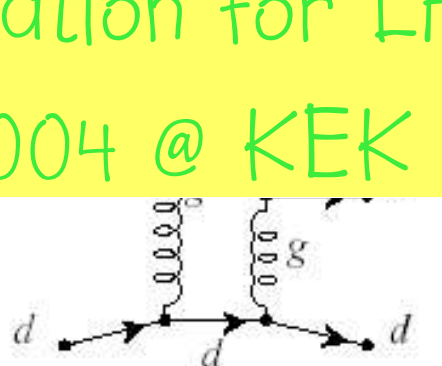
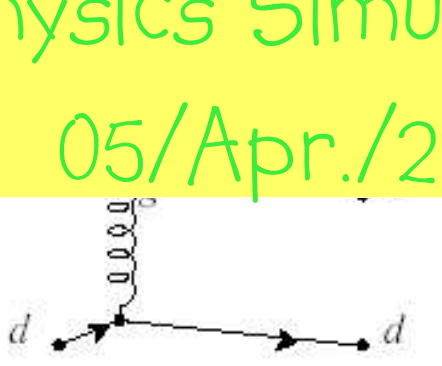
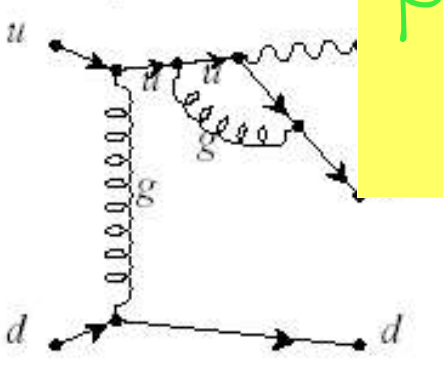
Graph 12



Y. Kurihara (KEK)  
 GRACE Group  
 Physics Simulation for LHC  
 05/Apr./2004 @ KEK

Graph 13

Graph 16



# Motivation

 LHC Experimental requirement

New Particle Search/Precision Measurement

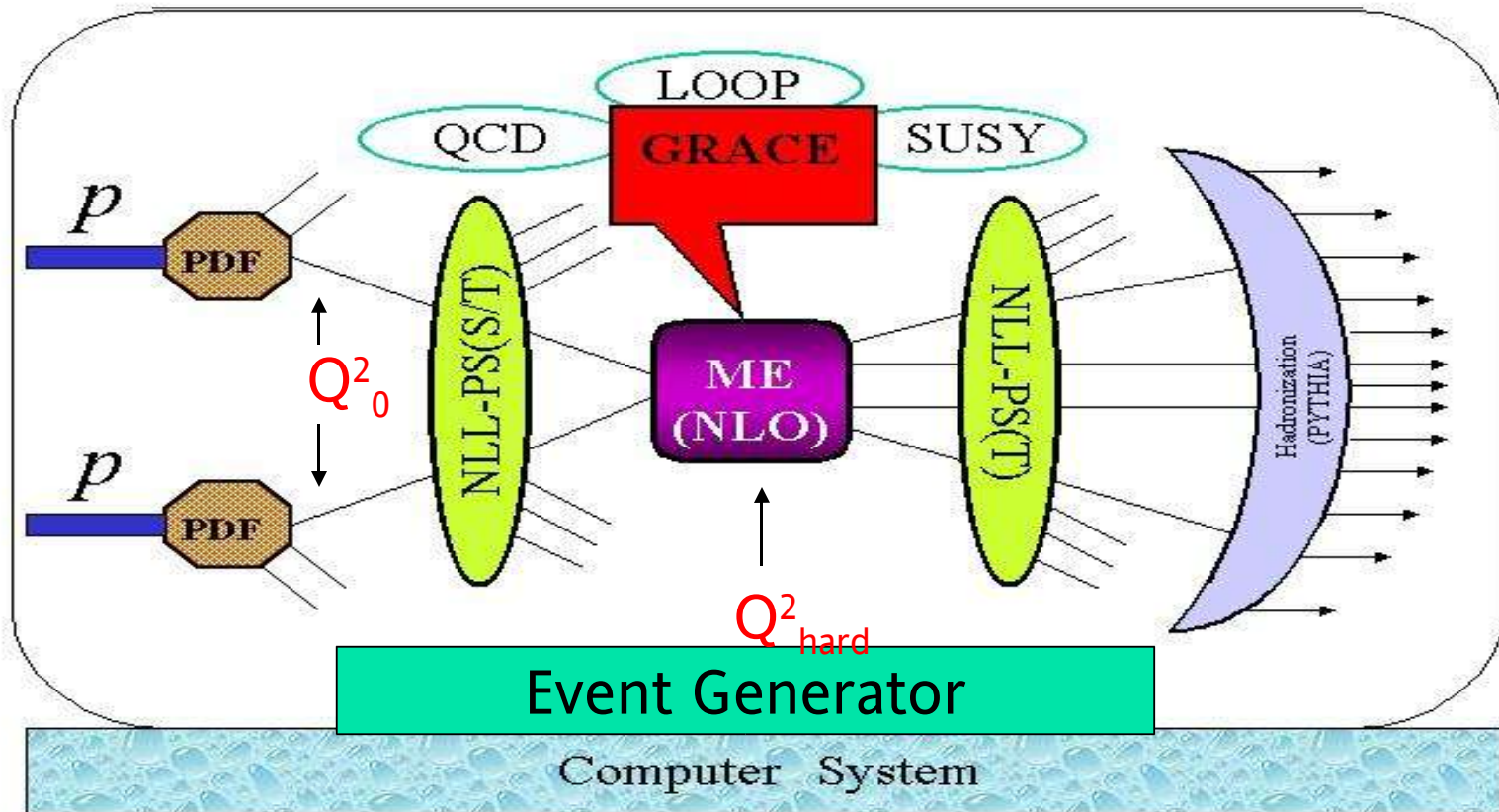
LO-QCD Event generator+K-factor



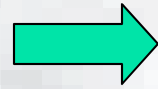
Obviously not enough!

We need  
NLO Event generator!

# Grand Design



# Difficulties

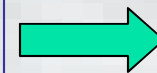


# Solutions

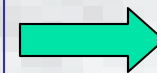
- Large number of diagrams
- Numerical instability due to a collinear singularity
- Double counting between ME and PDF/PS
- Negative weight



## GRACE



## LL-subtraction + Parton Shower



## Improved SPRING

# NLO matrix elements

## • $2 \rightarrow 2$ processes

- Drell-Yan process G. Altarelli et al. (1979)
- Two-jet production (1986~1992)
- Prompt photon production P. Aurenche et al. (1988)
- Z-pair production B. Mele et al. (1991)
- Heavy quark production M.L. Mangano et al. (1992)
- W-pair production S. Frixione et al. (1993)
- V + jet W.T. Giele et al. (1993)

## • $2 \rightarrow 3$ processes

- Three jets Z. Bern et al. (1993)
- V+two-jet J. Campbell et al. (2003)

$$pp \rightarrow W + 3 \text{ jets}$$

31 processes (initial 1 generation,  
final 2 generations)

Tree 730 Feynman Graphs

1 loop 20915 Feynman Graphs

$$pp \rightarrow W + 4 \text{ jets}$$

96 processes (initial 1 generation,  
final 2 generations)

Tree 9716 Feynman Graphs

1 loop 414906 Feynman Graphs



Automatic calculation by GRACE

# GRACE author list



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*d) Kogakuin University, Nishi-Shinjuku 1-24, Shinjuku, Tokyo 163-8677, Japan*

*e) Seikei University, Musashino, Tokyo 180-8633, Japan*

*f) Rikkyo University, 3-34-1 Nishi-Ikebukuro, Toshima, Tokyo 171-8501, Japan*

# GRACE Family

- GRACE/SM

Tree diagrams/Standard Model

T. Ishikawa, et al., **GRACE manual (v1.0)**, KEK Report 92-19

F. Yuasa, et al., Prog. Theor. Phys. (Suppl.) **138** (2000) 18

- GRACE/SUSY (v2.2.0)

Tree diagrams/MSSM

M. Kuroda, hep-ph/9902349

J. Fujimoto, et al., Compt. Phys. Commun. **153** (2003) 106

Programs are available from  
<http://minami-home.kek.jp>



- GRACE-loop

1-loop diagrams/Standard Model (w/ NLG)

$e^+e^- \rightarrow \nu\nu h$ : G. Belanger, et al., Phys. Lett. B 559 (2003) 252

NLG : G. Belanger, et al., hep-ph/9907406

$e^+e^- \rightarrow t\bar{t}h, Zhh$  (G. Belanger's talk)

- GRACE/SUSY-loop

1-loop diagrams/MSSM(w/ NLG)

Under development

$e^+e^- \rightarrow \chi_1^+\chi_1^-$ : T. Kon, et al., LoopFest II, 15/May/2003 @ BNL

- GRACE/NLO-QCD

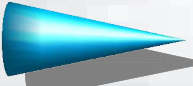
1-loop diagrams/QCD

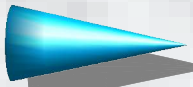
Under development

Y. Kurihara, et al., Nucl. Phys. B 654 (2003) 301

# ● Parton Shower

## ➤ PYTHIA/HERWIG

Q<sup>2</sup> evolution/x-distribution  PDF (Parton distribution function)

Soft-Collinear jets  (backward) Parton Shower

PS is used as a model to generate jets

## ➤ Our strategy

Q<sup>2</sup> evolution/x-distribution/Soft-Collinear jets

 LL/NLL Parton Shower (forward)

PS is legitimate child of pQCD  
(no tunable parameters)

# Parton Shower

DGLAP Equation

$$\frac{dD(x, Q^2)}{d\ln Q^2} = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(x/y) D(y, Q^2)$$

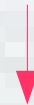
Splitting function ← pQCD



$$D(x, Q^2) = \Pi(Q^2, Q_s^2) D(x, Q_s^2) + \frac{\alpha}{2\pi} \int_{Q_s^2}^{Q^2} \frac{dK^2}{K^2} \Pi(Q^2, K^2) \int_x^{1-\epsilon} \frac{dy}{y} P(y) D(x/y, K^2)$$

$$\Pi(Q^2, Q_s^2) = \exp\left(-\frac{\alpha}{2\pi} \int_{Q_s^2}^{Q^2} \frac{dK^2}{K^2} \int_0^{1-\epsilon} dx P(x)\right)$$

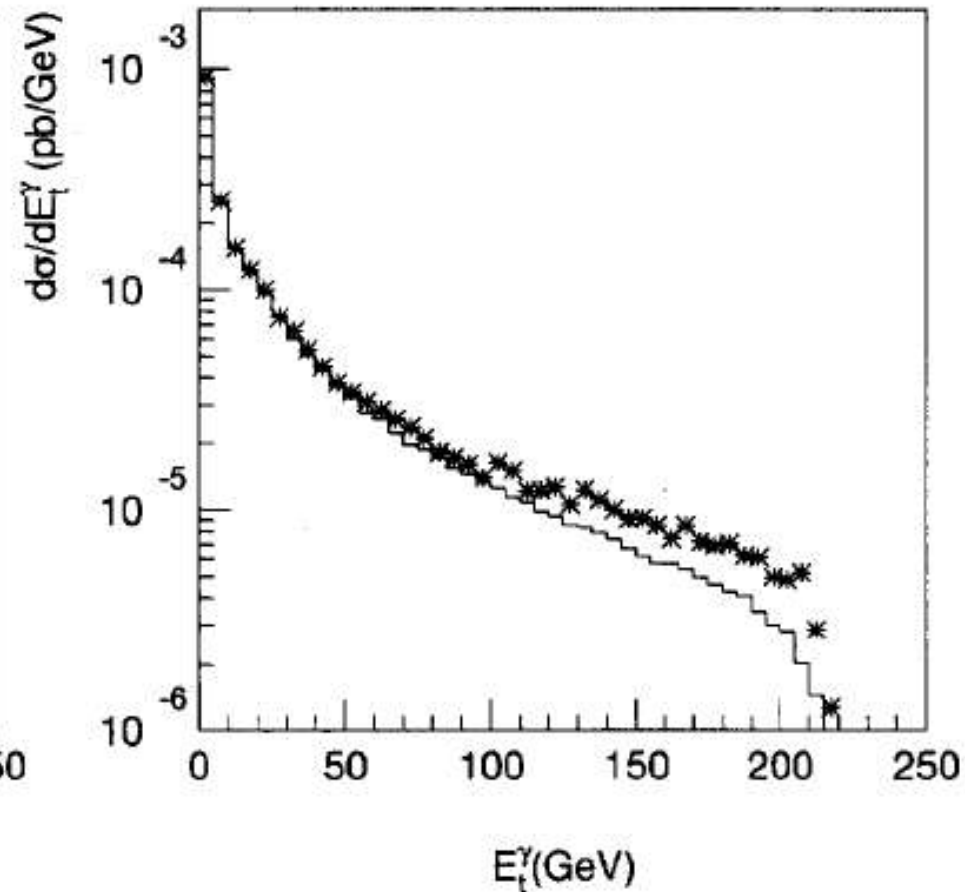
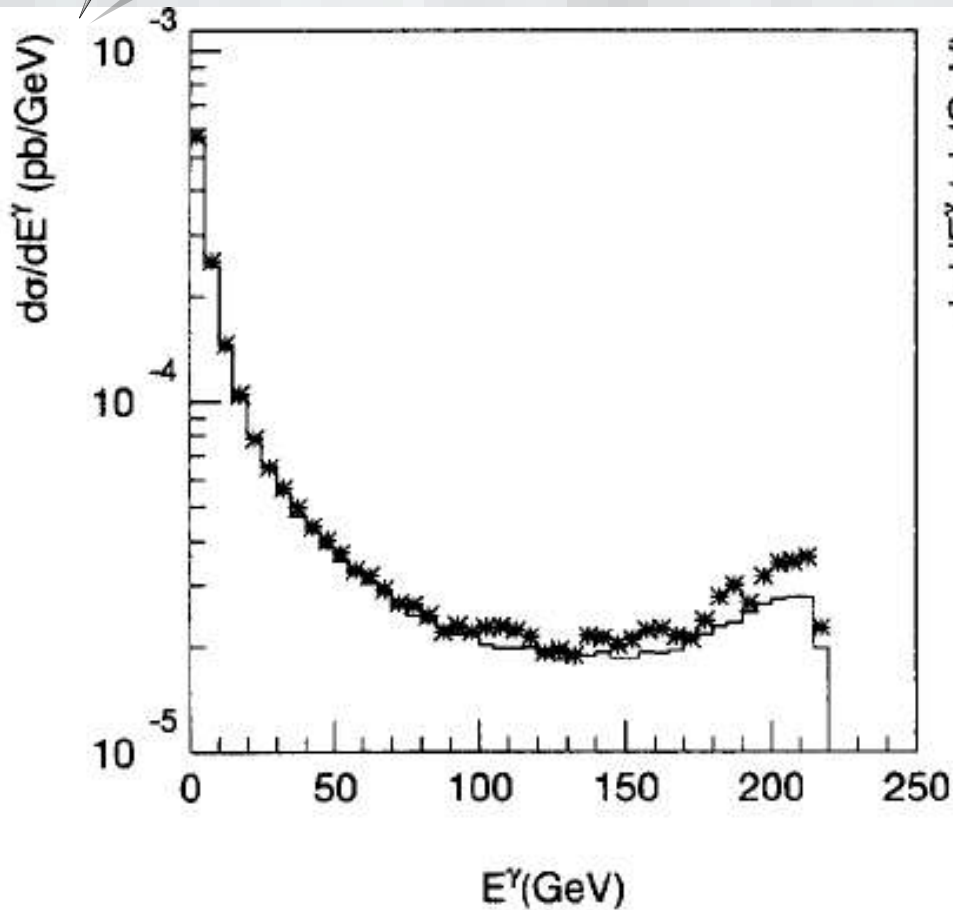
Sudakov Factor



non-branch provability

# • Example in QED (annihilation)

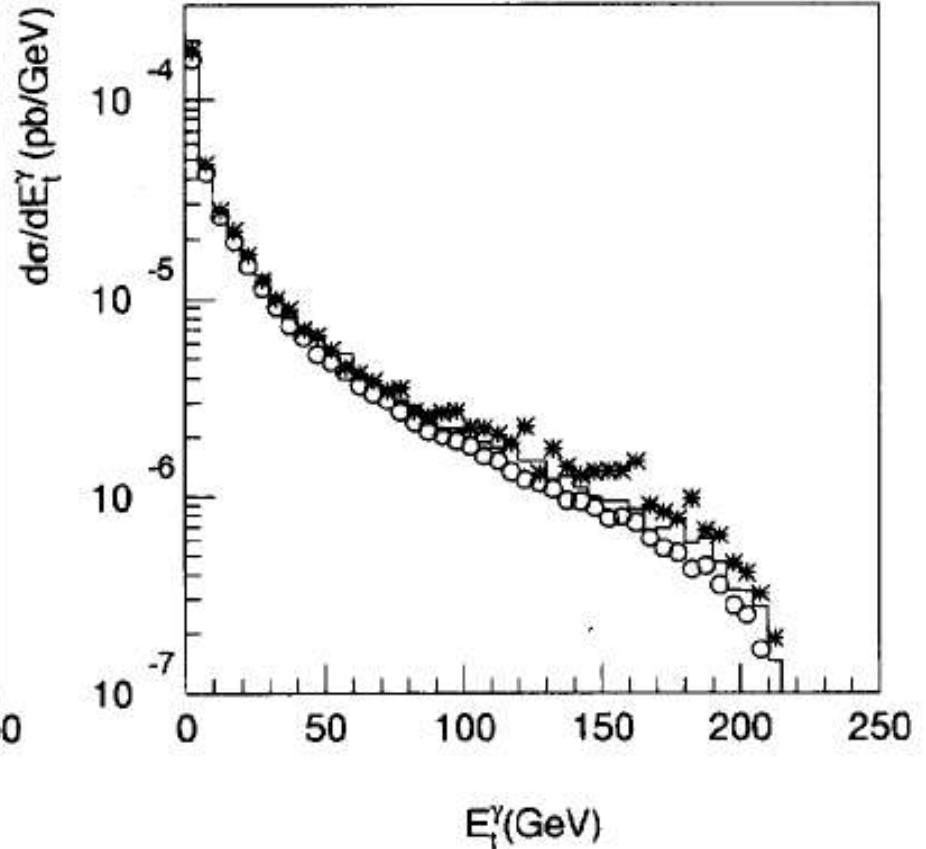
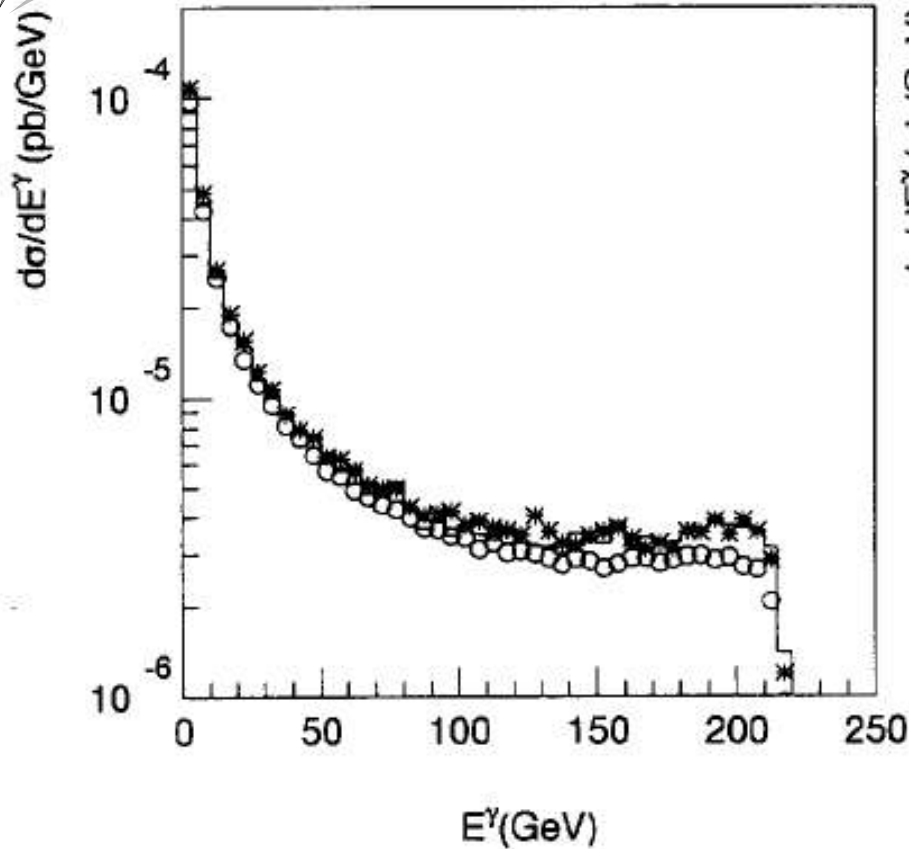
$$e^+e^- \rightarrow ZH + \gamma$$



# • Example in QED (annihilation)

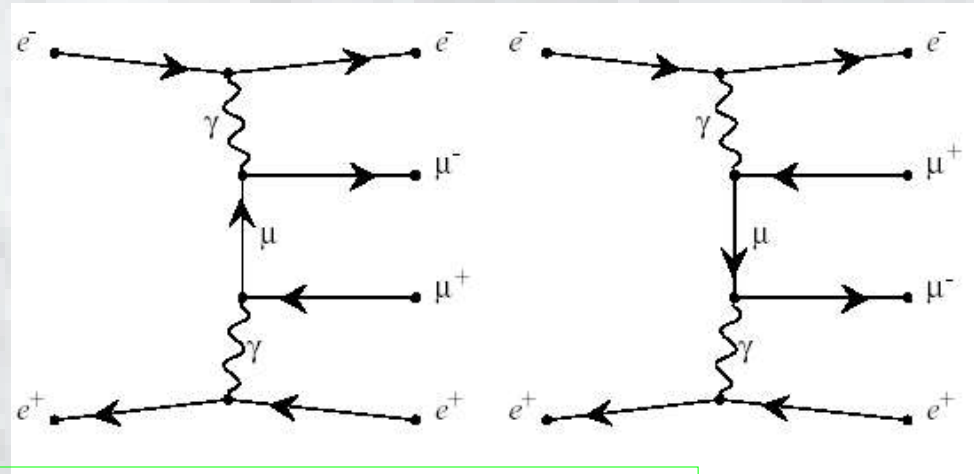
$$e^+e^- \rightarrow ZH + \gamma\gamma$$

- Zh $\gamma\gamma$  ME
- \* ZH ME +  $\gamma\gamma$ PS
- Zh $\gamma\gamma$  ME+PS(soft)



# • Example in QED (fusion)

$$e^+e^- \rightarrow e^+e^-\mu^+\mu^-$$



★ Soft correction formula

$$\sigma_{\text{soft}} = \sigma_0(s) \left\{ 1 + \frac{\alpha}{\pi} \left[ -2l(L-1) + \frac{3}{2}(L-1) \right] \right\}$$

★  $O(\alpha)$  correction

$$2\text{Re}F_1 + \delta_s \rightarrow \frac{\alpha}{\pi} \left( -2l(L_t - 1) + \frac{3}{2}L_t - 2 \right)$$

$$L_t = \ln(-t/m_e^2)$$

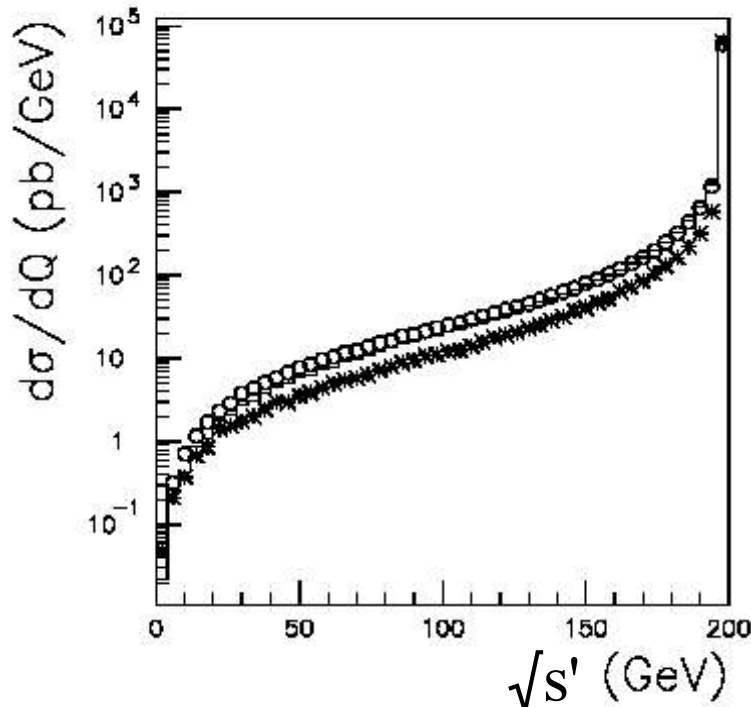
Factorization  
energy scale

# $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ : Total cross sections



$E_{CM}$ (GeV)	$\sigma_0$ (nb)	$\sigma_{sf}$ (nb)	$\sigma_{QEDPS}$ (nb)	$\sigma_{BDK}$ (nb)
20	97.0(1)	96.3(1)	96.0(2)	96.0(1)
40	137.5(3)	136.3(1)	135.9(3)	135.9(1)
100	202.8(4)	201.0(2)	200.5(4)	200.5(2)
200	262.0(6)	259.8(3)	259.1(6)	258.8(2)

F. A. Berends, P. H. Daverveldt and R. Kleiss, Nucl. Phys. **B253** (1985), 412.



- GRACE+PS
- \* GRACE+SF
- BDK

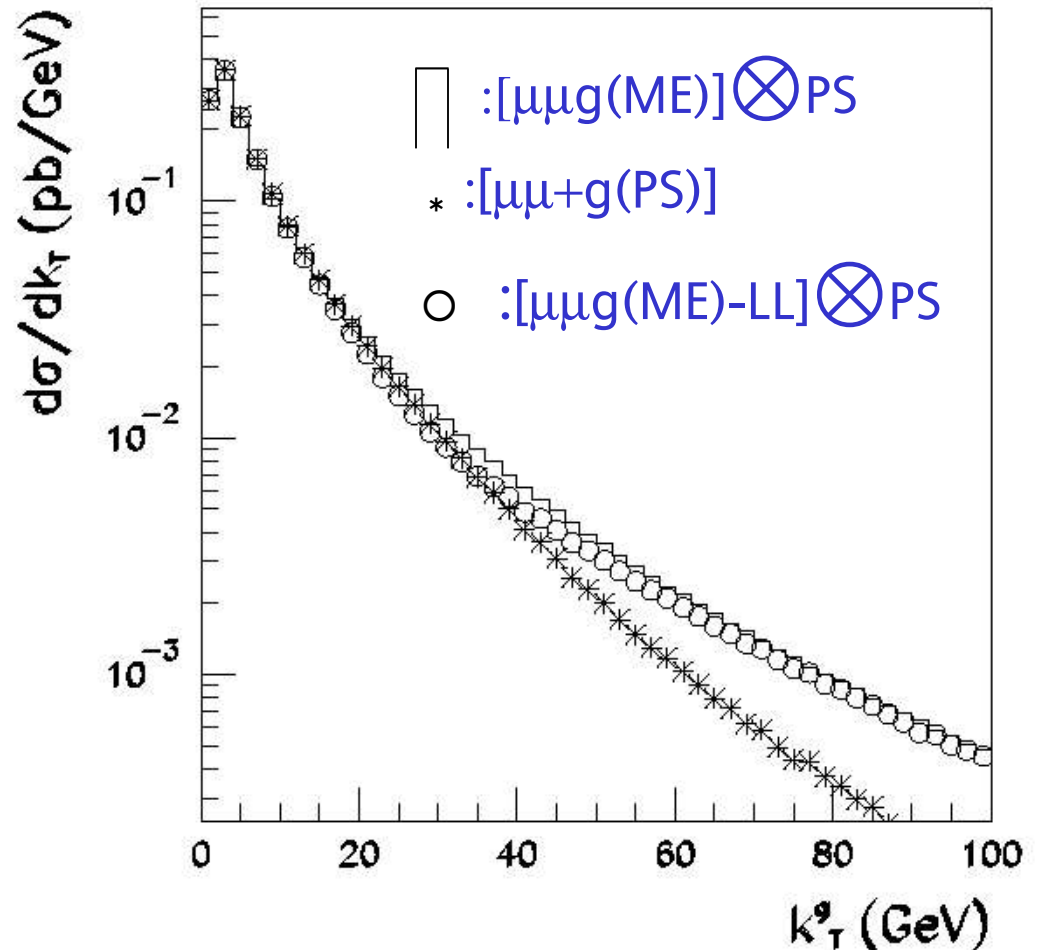
Y. Kurihara, J. Fujimoto, Y. Shimizu, K. Kato, K. Tobimatsu, T. Munehisa, Prog. Theor. Phys. **103**, 1199 (2000)

# Drell-Yan process



## $k_T^g$ Test

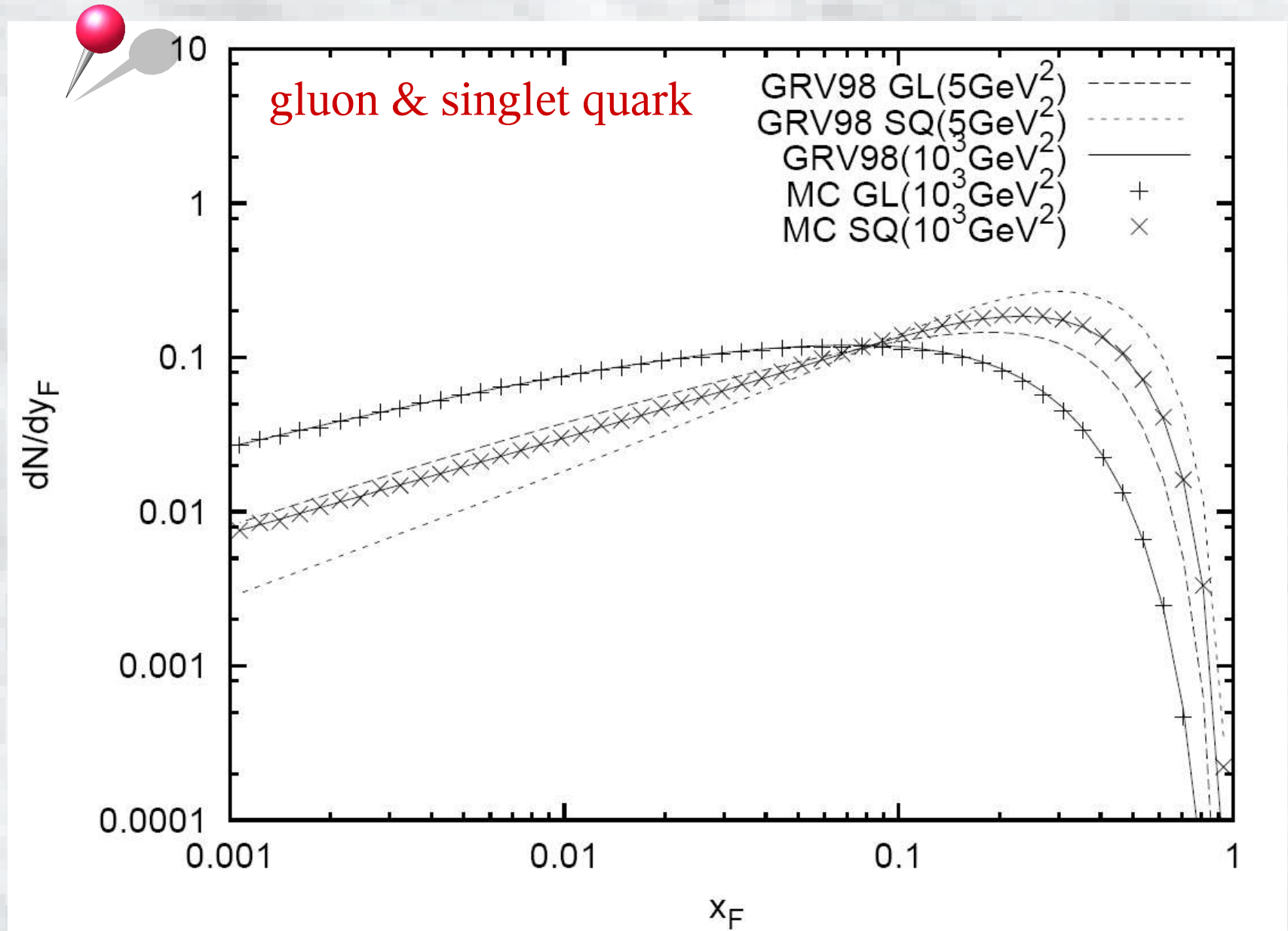
- Process :  
 $u\bar{u} \rightarrow \mu^+\mu^- (+\text{gluon})$   
in pp collision
- Cuts:  
 $\sqrt{s_{\mu\mu}} > 40 \text{ GeV}$   
 $k_T^g > 1 \text{ GeV}$





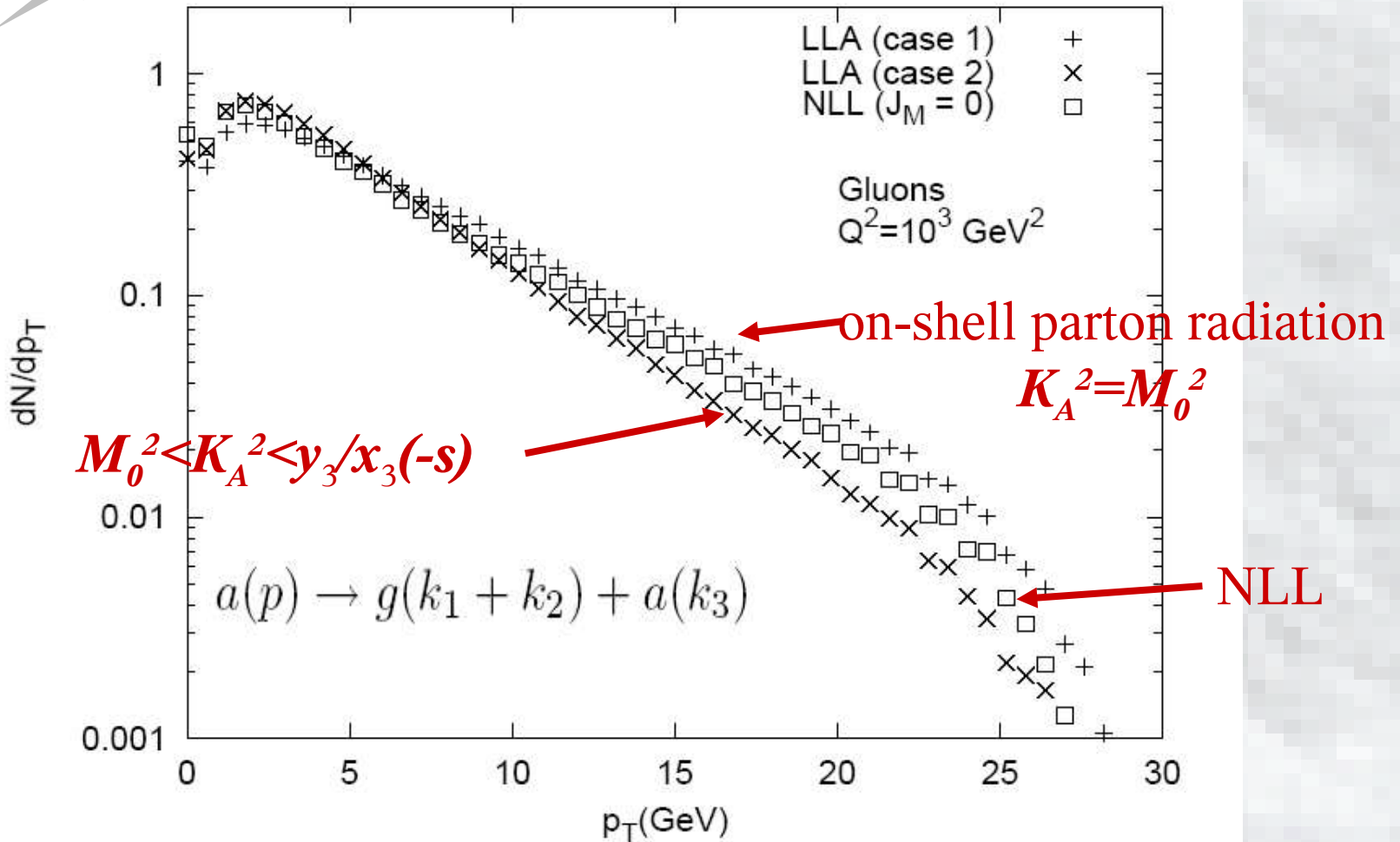
# ✓ NLL Parton Shower

H. Tanaka, PTP 110 (2003) 963.





# $p_T$ distribution Gluon

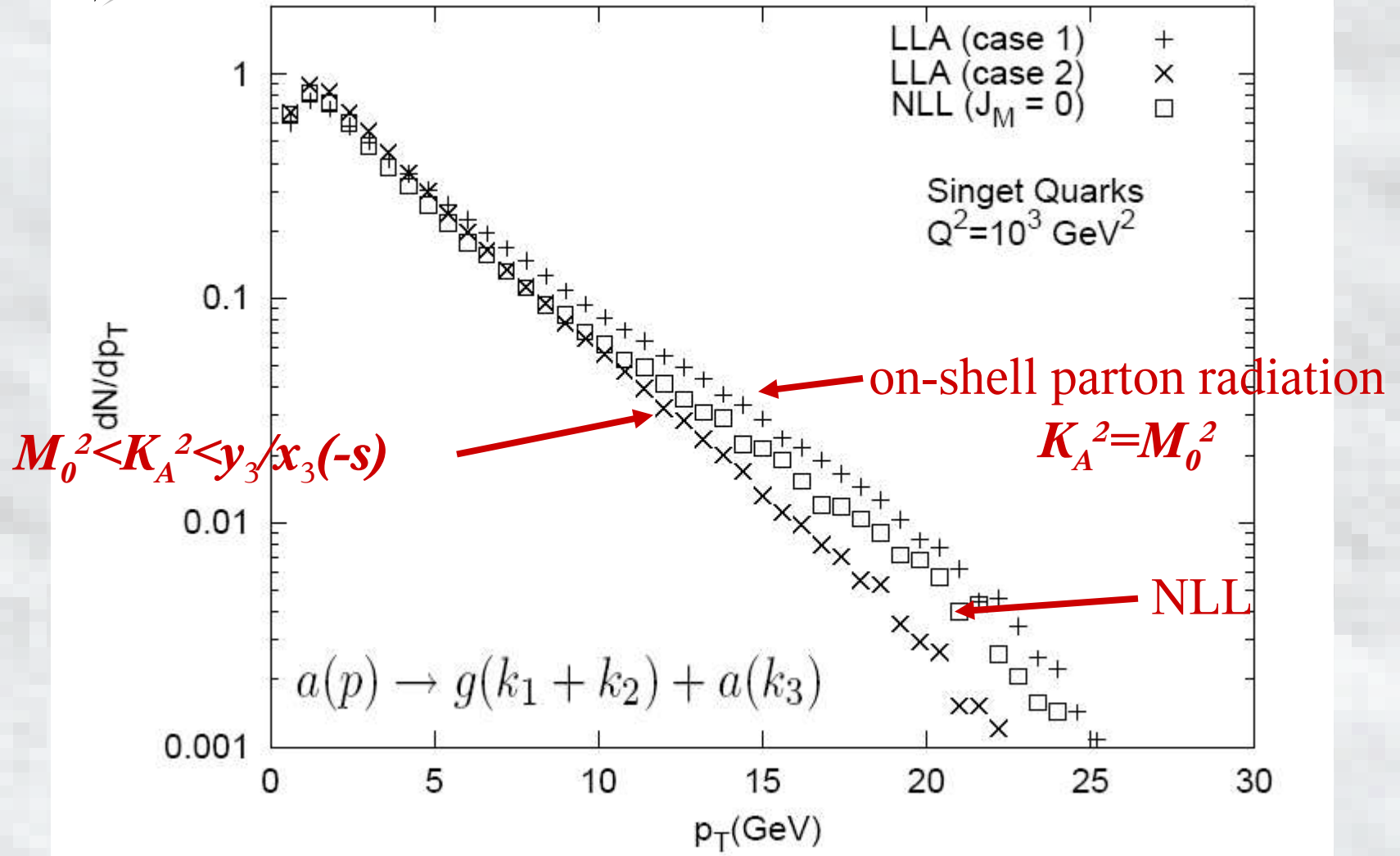


$$p_T^2 = x_3 y_3 \left[ p^2 + \frac{-s}{x_3} - \frac{K_A^2}{y_3} \right]$$

$$K_A^2 = (k_1 + k_2)^2, s = k_3^2 \text{ and } p_T^2 = \vec{k}_{3T}^2$$

$$y_3 = 1 - x_3$$

# $p_T$ distribution : singlet quarks



# Double Counting

No IR-divergence

$$\sigma_{\text{NLO}} = [\sigma_{\text{tree}} (1 + \delta_V + \delta_{s/c}) + \sigma_{\text{vis}}] \otimes \text{PDF/PS}$$

$1/\epsilon_{\text{IR}}^2, 1/\epsilon_{\text{IR}}$  cancellation

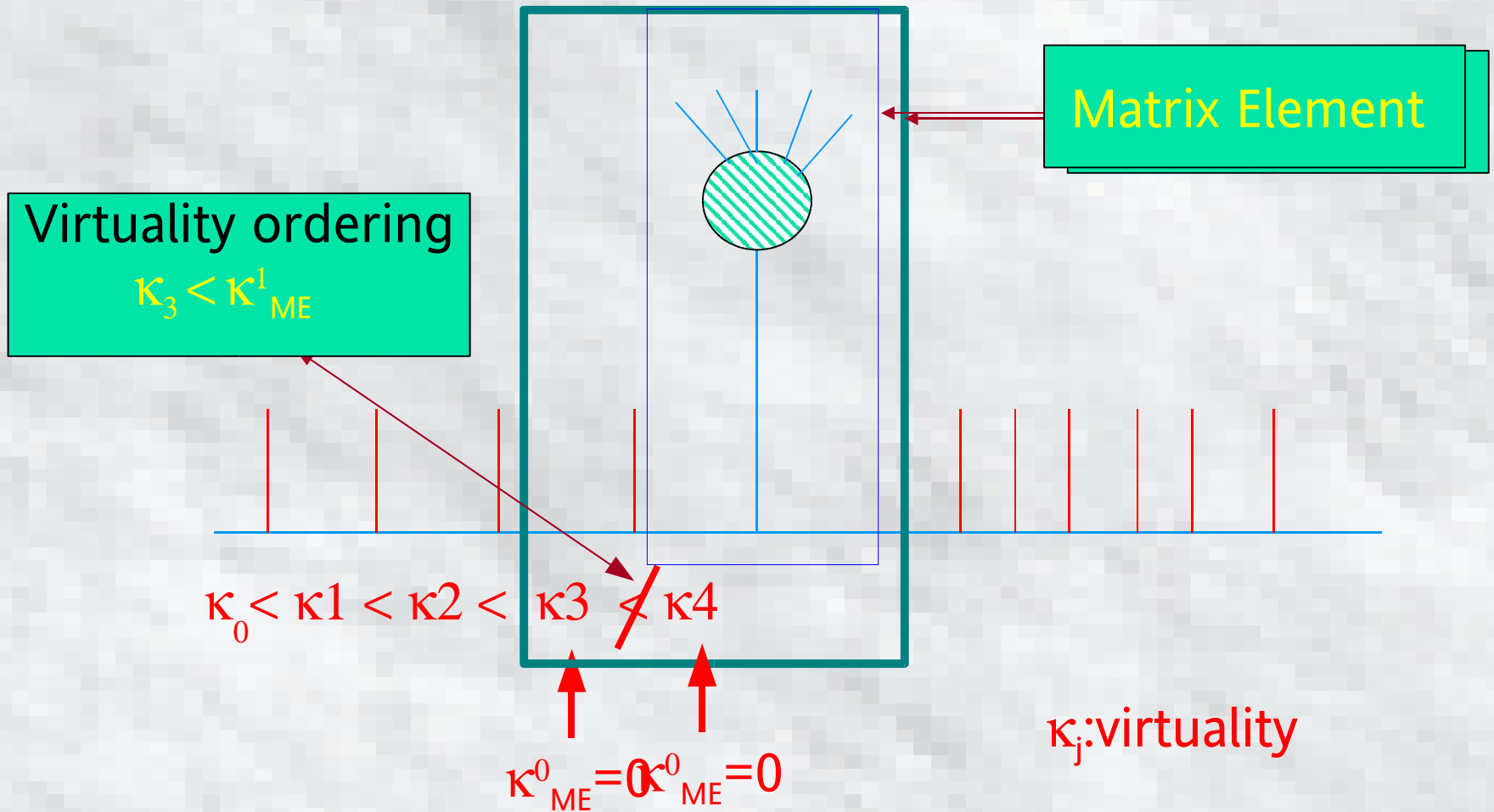
PDF/Parton Shower

$$\frac{1}{\epsilon_{\text{IR}}} f_c \frac{\alpha_s}{2\pi} P(x)$$

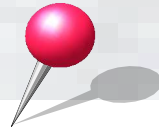
$P(x)$  : Splitting function

Space/time dimension :  $d=4+2\epsilon_{\text{IR}}$

# Double Counting Rejection

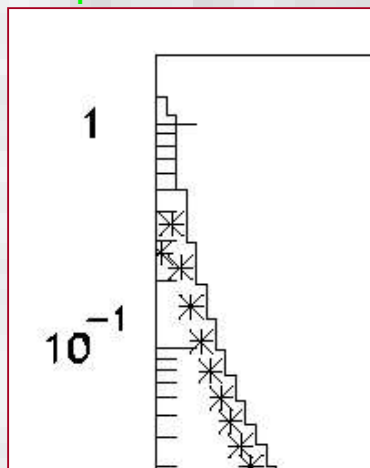


# Drell-Yan process

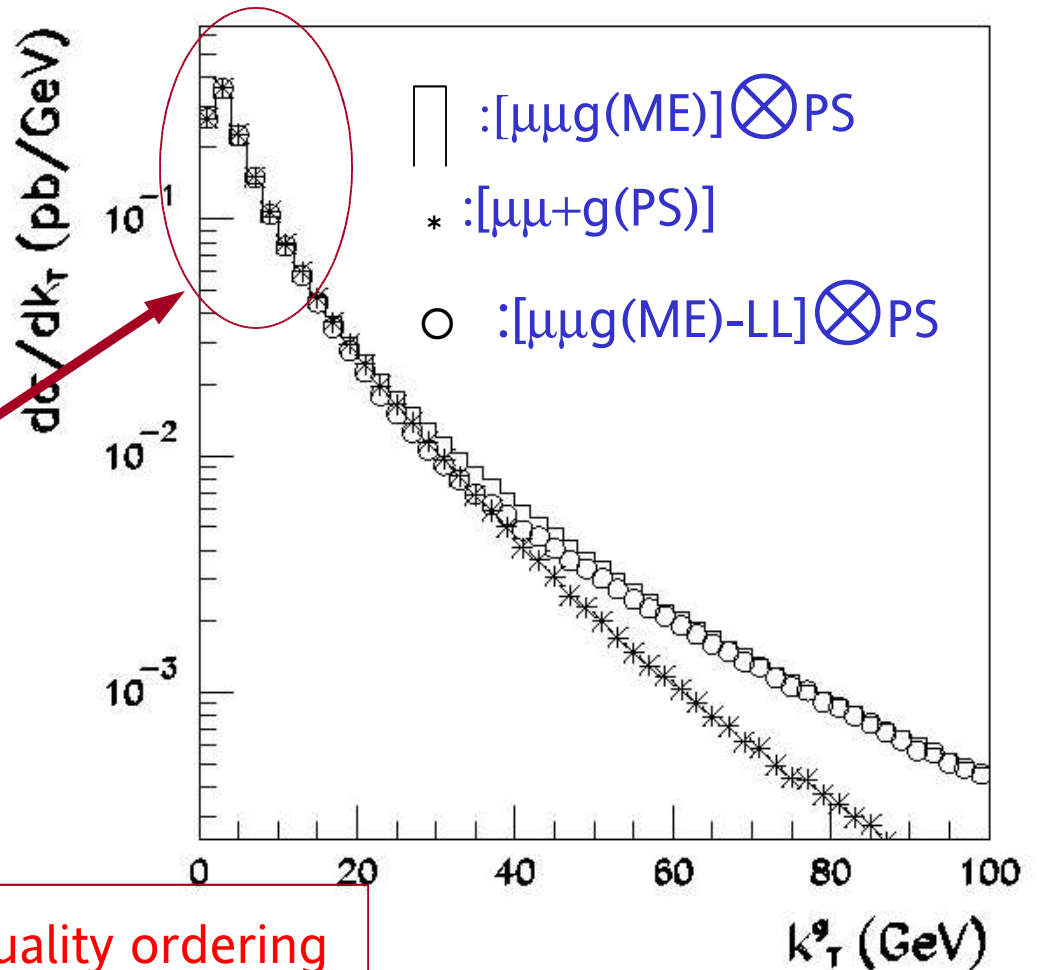


## $k_T^g$ Test

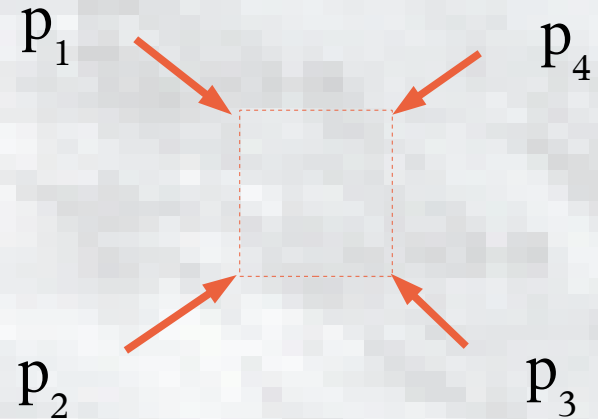
- Process :  
 $u\bar{u} \rightarrow \mu^+\mu^- (+\text{gluon})$   
in pp collision
- Cuts:  
 $\sqrt{s}_{\mu\mu} > 40 \text{ GeV}$   
 $k_T^g > 1 \text{ GeV}$



w/o virtuality ordering



# Box Integral



$$J_{(4)}(s, t, p_1^2, p_2^2, p_3^2, p_4^2; n_x, n_y, n_z) = \frac{\Gamma(2 - \epsilon_{IR})}{(4\pi)^2 (4\pi\mu_R^2)^{\epsilon_{IR}}} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x^{n_x} y^{n_y} z^{n_z}}{D^{2-\epsilon_{IR}}},$$

$$D = -s xz - t yw - p_1^2 xy - p_2^2 yz - p_3^2 zw - p_4^2 xw - i0,$$

$$w = 1 - x - y - z,$$

$$s = (p_1 + p_2)^2,$$

$$t = (p_1 + p_4)^2.$$

# All on-shell (massless) external legs

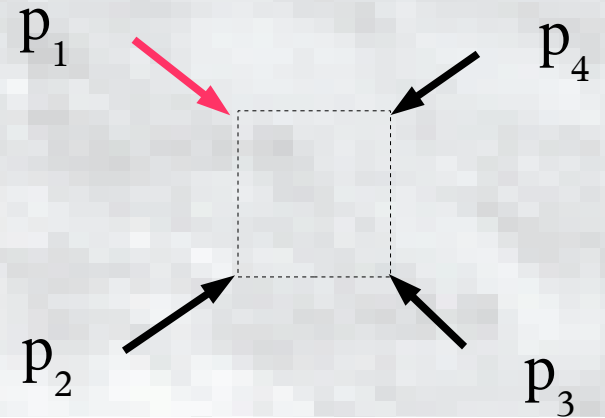
$$\begin{aligned}
 J_4(s, t; 0, 0, 0, 0; n_x, n_y, n_z) &= \frac{1}{(4\pi)^2 s t} B(n_x + \varepsilon_{IR}, n_y + n_z + \varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR}) \Gamma(1 - \varepsilon_{IR}) \\
 \times &\left[ \left( \frac{-\tilde{t}}{4\pi\mu_R^2} \right)^{\varepsilon_{IR}} \left( \frac{-t}{s} \right)^{n_x} \frac{B(1 + n_z, n_x + n_y + \varepsilon_{IR})}{\Gamma(n_x + \varepsilon_{IR})} \right. \\
 \times &{}_2F_1 \left( 1 + n_x, n_x + n_y + \varepsilon_{IR}, 1 + n_x + n_y + n_z + \varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}} \right) \\
 + &\left( \frac{-\tilde{s}}{4\pi\mu_R^2} \right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \left( \frac{-s}{t} \right)^l \frac{(-1)^l}{\Gamma(l + \varepsilon_{IR}) (n_x - l)!} B(1 + n_y, l + n_z + \varepsilon_{IR}) \\
 \times &{}_2F_1 \left( 1 + l, l + n_z + \varepsilon_{IR}, 1 + l + n_y + n_z + \varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}} \right) \left. \right],
 \end{aligned}$$

## Scalar Integral

$$\begin{aligned}
 J_{(4)}(s, t; 0, 0, 0, 0; 0, 0, 0) &= \frac{1}{(4\pi)^2 s t} \frac{B(\varepsilon_{IR}, \varepsilon_{IR}) \Gamma(1 - \varepsilon_{IR})}{\varepsilon_{IR}} \\
 \times &\left[ \left( \frac{-\tilde{s}}{4\pi\mu_R^2} \right)^{\varepsilon_{IR}} {}_2F_1 \left( 1, \varepsilon_{IR}, 1 + \varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}} \right) + \left( \frac{-\tilde{t}}{4\pi\mu_R^2} \right)^{\varepsilon_{IR}} {}_2F_1 \left( 1, \varepsilon_{IR}, 1 + \varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}} \right) \right]
 \end{aligned}$$



# One off-shell box integral



$$\begin{aligned}
 J_4(s, t; p_1^2, 0, 0, 0; n_x, n_y, n_z) &= \\
 & \frac{\Gamma(2 - \varepsilon_{IR})}{(4\pi)^2 (4\pi\mu_R^2)^{\varepsilon_{IR}}} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x^{n_x} y^{n_y} z^{n_z}}{(-xz s - y(1-x-y-z)t - p_1^2 xy - i0)^{2-\varepsilon_{IR}}} \\
 &= \frac{1}{(4\pi)^2 s t} B(n_x + \varepsilon_{IR}, n_y + n_z + \varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR}) \Gamma(1 - \varepsilon_{IR}) \\
 & \times \left[ \left( \frac{-\tilde{t}}{4\pi\mu_R^2} \right)^{\varepsilon_{IR}} \left( \frac{-t}{s} \right)^{n_x} \frac{B(1 + n_z, n_x + n_y + \varepsilon_{IR})}{\Gamma(n_x + \varepsilon_{IR})} \mathcal{I}^{(1)} \right. \\
 & \left. + \left( \frac{-\tilde{s}}{4\pi\mu_R^2} \right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \frac{(-1)^l B(1 + n_y, l + n_z + \varepsilon_{IR})}{\Gamma(l + \varepsilon_{IR}) (n_x - l)!} \mathcal{I}_l^{(2)} \right]
 \end{aligned}$$

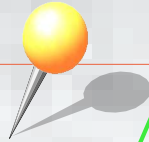
$$u = -s - t + p_1^2$$


$$\mathcal{I}^{(1)} = B(1 + n_z, n_x + n_y + \varepsilon_{IR}) {}_2F_1 \left( 1 + n_x, n_x + n_y + \varepsilon_{IR}, 1 + n_x + n_y + n_z + \varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}} \right)$$

$$\begin{aligned} \mathcal{I}_l^{(2)} &= \sum_{k_1=0}^{n_z} n_z C_{k_1} \left( \frac{s}{p_1^2 - s} \right)^{n_y + k_1} \sum_{k_2=0}^{n_y + k_1} n_y + k_1 C_{k_2} (-1)^{n_y + k_2} \left( \frac{-t}{s} \right) \\ &\times \int_0^1 dw \left( 1 + \frac{\tilde{u}}{\tilde{s}} w \right)^{-(l+1)} \left( 1 + \frac{\tilde{t} + \tilde{u}}{\tilde{s}} w \right)^{k_2 + l - 1 + \varepsilon_{IR}} \\ &= \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y + k_1} n_z C_{k_1} n_y + k_1 C_{k_2} (-1)^{k_1 + k_2} \left( \frac{s}{p_1^2 - s} \right)^{n_y + k_1} \frac{1}{l + k_2 + \varepsilon_{IR}} \left( 1 + \frac{u}{t} \right)^l \\ &\times \left[ {}_2F_1 \left( 1 + l, l + k_2 + \varepsilon_{IR}, 1 + l + k_2 + \varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}} \right) \right. \\ &\left. - \left( \frac{\tilde{p}_1^2}{\tilde{s}} \right)^{l + k_2 + \varepsilon_{IR}} {}_2F_1 \left( 1 + l, l + k_2 + \varepsilon_{IR}, 1 + l + k_2 + \varepsilon_{IR}, -\frac{\tilde{u} \tilde{p}_1^2}{\tilde{t} \tilde{s}} \right) \right], \end{aligned}$$



# Status of GRACE NLO Generator



## Already tested

- + Drell-Yan process
- + W production



## Under development

- + Prompt photon
- + V+1 jet

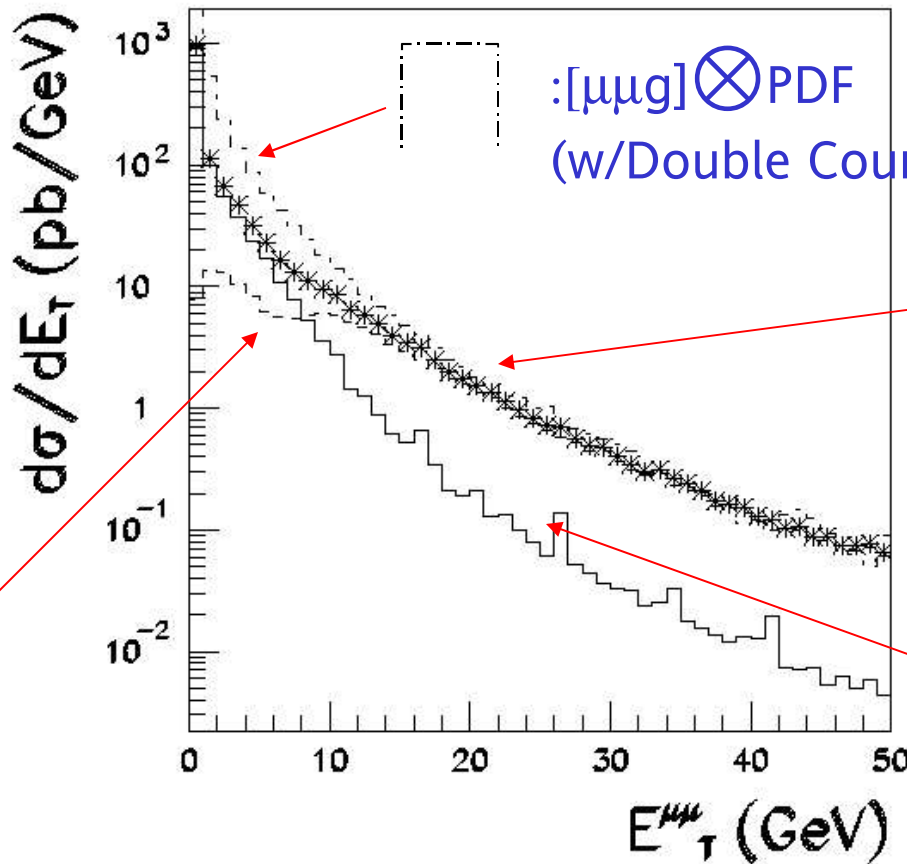


## Future plan

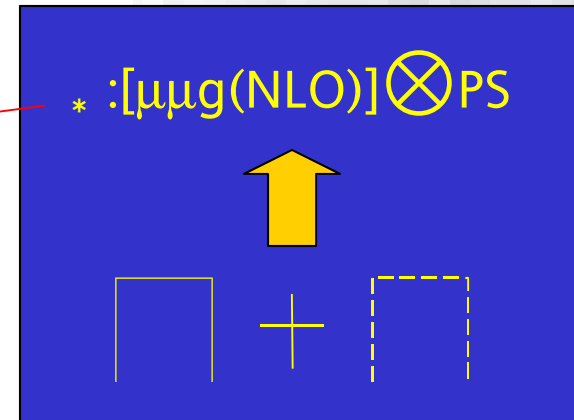
- + V+2 jets,.....

# Drell-Yan Process @ NLO

$E_T$  of  $\mu$ -pair system



$:[\mu\mu g] \otimes \text{PDF}$   
(w/Double Counting)



$:[\mu\mu(t+v+c)]/PS$

$:[\mu\mu g\text{-LL}] \otimes PS$

# Summary

## (1) Matrix Elements

- Automatic generation by GRACE

## (2) Parton Shower

- $x$ -deterministic PS at LL
- NLL-PS is available

## (3) Soft/Collinear treatment

- (N)LL-subtraction method

## (4) Application

- Drell-Yan process/w production
- $V+1\text{jet}, 2\text{jet}$