NLO Calculation in GRACE

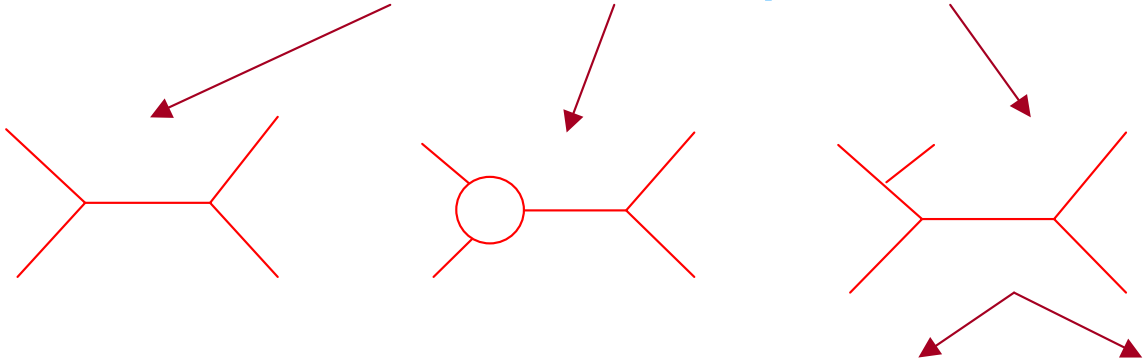
Yoshimasa Kurihara
(KEK)

Les Houches 2003
QCD/MC working group
03/June/2003@Les Houches

Y. Kurihara et al., Nucl. Phys B 645 (2003) 301

NLO Cross sections

$$\sigma_{NLO}=\sigma_{tree}+\sigma_{loop}+\sigma_R$$



$$=\sigma_{tree}(1+\delta_V+\delta_{s/c})+\sigma_{vis}$$

δ_V : Virtual (loop) correction

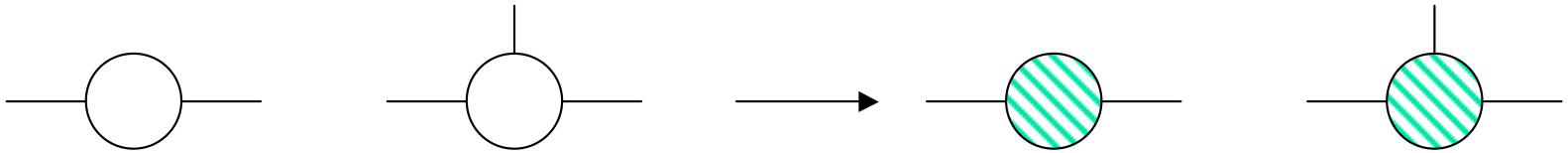
$\delta_{s/c}$: Soft/Collinear correction

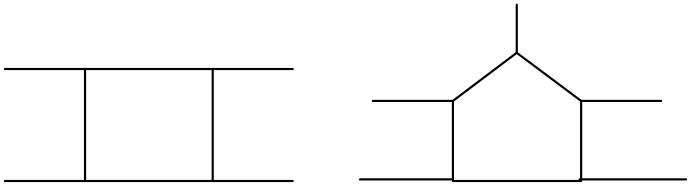
σ_{vis} : Visible jet cross section

Matrix Elements: Loop

- Loop diagrams

$M_V(1,2 \rightarrow 1,2\ldots,n)$: Effective vertices (up to three point)

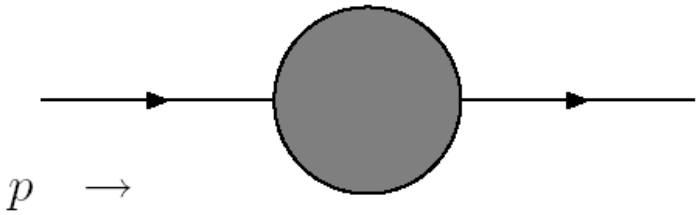




: Numerical calc. by CHANEL

Matrix Elements: Effective coupling

- Quark self-energy

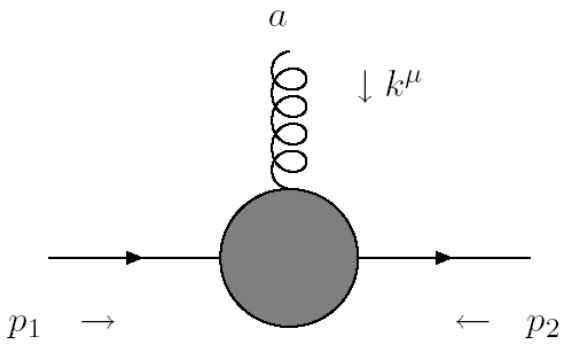


$$= \Sigma(p^2)$$

on-/off-shell	self-energy
$p^2 \neq 0$	$\Sigma(p^2) = C_F \frac{\alpha_s}{4\pi}(1 - \ln \frac{-p^2}{\mu^2})$
$p^2 = 0$	$\Sigma(0) = C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon_{IR}}$

Matrix Elements: Effective coupling

- Quark-Quark-Gluon vertex



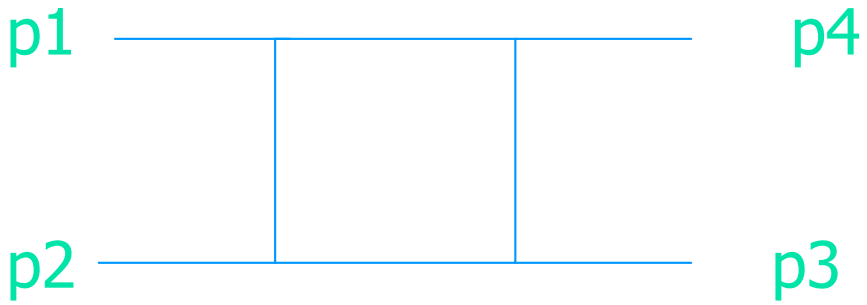
$$= \Lambda^a_\mu(p_1, p_2, k)$$

$$\begin{aligned}\Lambda_\mu(p_1,p_2,k) &= g\mathbf{T}^a(C_F - \frac{1}{2}C_G)\frac{\alpha_s}{4\pi}(\mathcal{F}_1^I\gamma^\mu + \mathcal{F}_2^I\frac{p_j^\mu \not{k}}{-p_j^2}) \\ &+ g\mathbf{T}^a\frac{1}{2}C_G\frac{\alpha_s}{4\pi}(\mathcal{F}_1^I\gamma^\mu + \mathcal{F}_2^I\frac{p_j^\mu \not{k}}{-p_j^2})\end{aligned}$$

$\mathcal{F}_1^I$	$\frac{2}{\varepsilon_{IR}} + L - 4$
$\mathcal{F}_2^I$	$\frac{4}{\varepsilon_{IR}} + 4L - 10$
$\mathcal{F}_1^{II}$	$-\frac{2}{\varepsilon_{IR}^2} + \frac{3-2L}{\varepsilon_{IR}} + \frac{\pi^2}{6} - L$
$\mathcal{F}_2^{II}$	$-\frac{2}{\varepsilon_{IR}^2} - \frac{2L}{\varepsilon_{IR}} + \frac{12+\pi^2-6L^2}{6}$

$$k^2 = 0,\ p_i^2 = 0,\ p_j^2 = q^2 \neq 0,\ \text{and}\ L = \ln\frac{-q^2}{\mu^2},$$

Loop Integrals (1)



$$J_4(s,t,m_1^2,m_2^2,m_3^2,m_4^2;n_x,n_y,n_z) =$$

$$\left(4\pi\mu^2\right)^{-\varepsilon_{IR}} \int_0^1 dx\ dy\ dz\ dw \frac{\delta(x+y+z+w-1)x^{n_x}y^{n_y}z^{n_z}}{D^{2-\varepsilon_{IR}}}$$

$$\begin{aligned} D &= -xzs - ywt - xym_1^2 - yzm_2^2 - zwm_3^2 - xwm_4^2 - i0 \\ m_i^2 &= p_i^2, \end{aligned}$$

Loop Integrals (2)

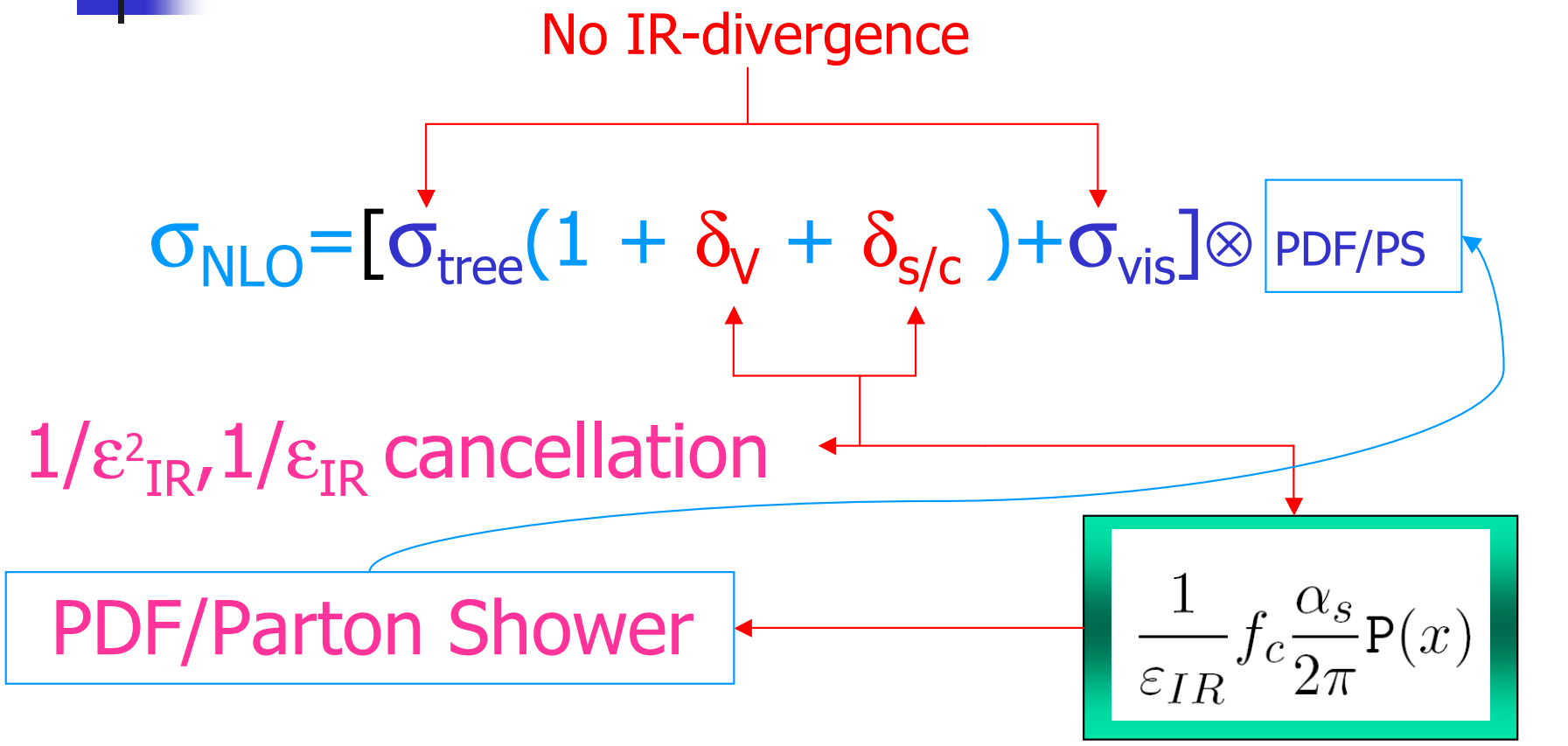
$$J_4(s,t,0,0,0,0;0,0,0) = \frac{1}{s\,t}\frac{B(\varepsilon_{IR},\varepsilon_{IR})}{\varepsilon_{IR}(1-\varepsilon_{IR})}$$

$$\times \left[\left(\frac{-\tilde{s}}{4\pi\mu^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{s}}\right)\right]$$

$$J_4(s,t,m_1^2,0,0,0;0,0,0) =$$
$$J_4(s,t,0,0,0,0;0,0,0) - \frac{1}{s\,t}\frac{B(\varepsilon_{IR},\varepsilon_{IR})}{\varepsilon_{IR}(1-\varepsilon_{IR})}\left(\frac{-\tilde{m}_1^2}{4\pi\mu^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}\tilde{m}_1^2}{\tilde{t}\tilde{s}}\right)\Bigg]$$

$${}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},z\right) = 1 - \ln(1-z)\varepsilon_{IR} - \mathrm{Li}(z)\varepsilon_{IR}^2 + O\left(\varepsilon_{IR}^3\right)$$

IR-behavior



P(x) : Splitting function

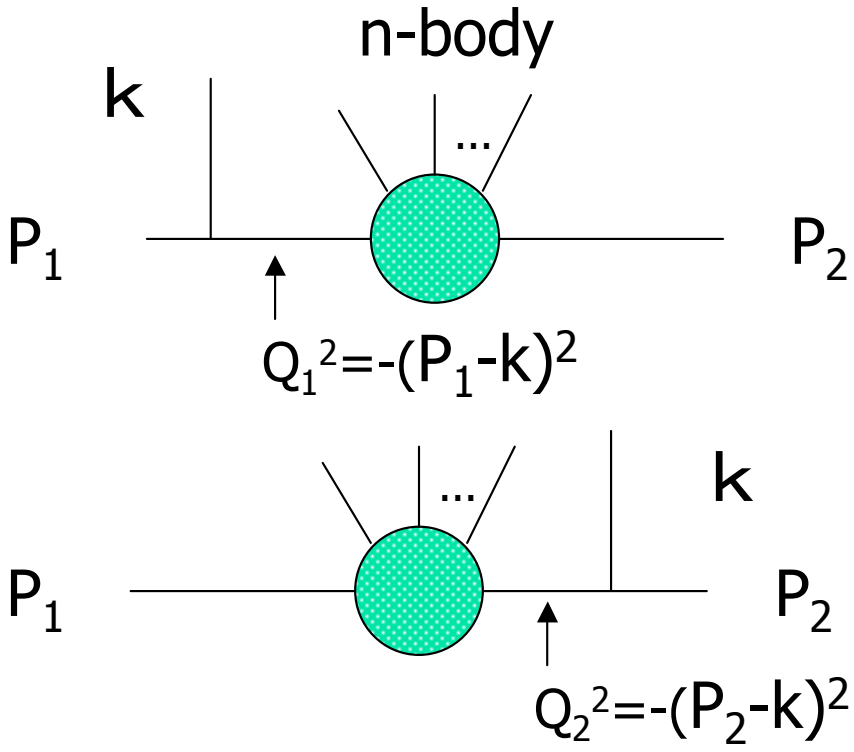
Space/time dimension : $d=4+2\varepsilon_{IR}$

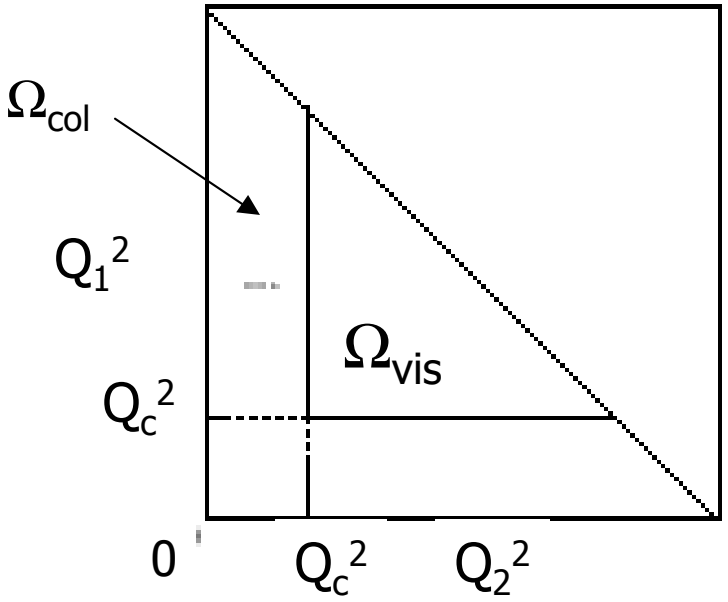
Soft/Collinear Treatment

- Subtraction method
- Phase-space slicing

S.Catani, M.M. Seymour, hep-ph/9605323
S. Frixione, B.R. Webber, hep-ph/0204244

W.B. Harris, J.F.Owens, Phys. Rev. D65 (2002) 094032





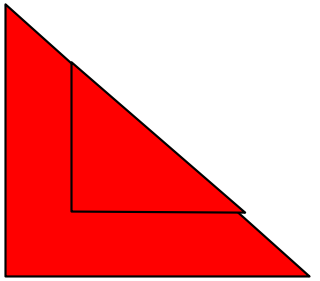
Collinear Cross Section

Cancelled with δ_V

$$\sigma_{coll} = \sigma_0(s)\frac{\alpha_s}{2\pi} f_c \left[\frac{2}{\varepsilon_{IR}^2} + \frac{2L-3}{\varepsilon_{IR}} - \frac{\pi^2}{2} + L^2\right]$$

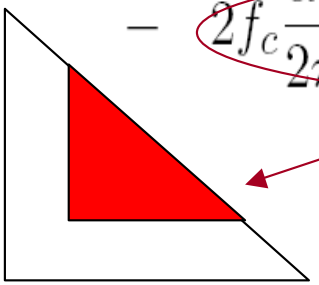
PDF/PS

$$+ 2\int_0^1 dx \sigma_0(xs)\phi(x,\varepsilon_{IR})$$



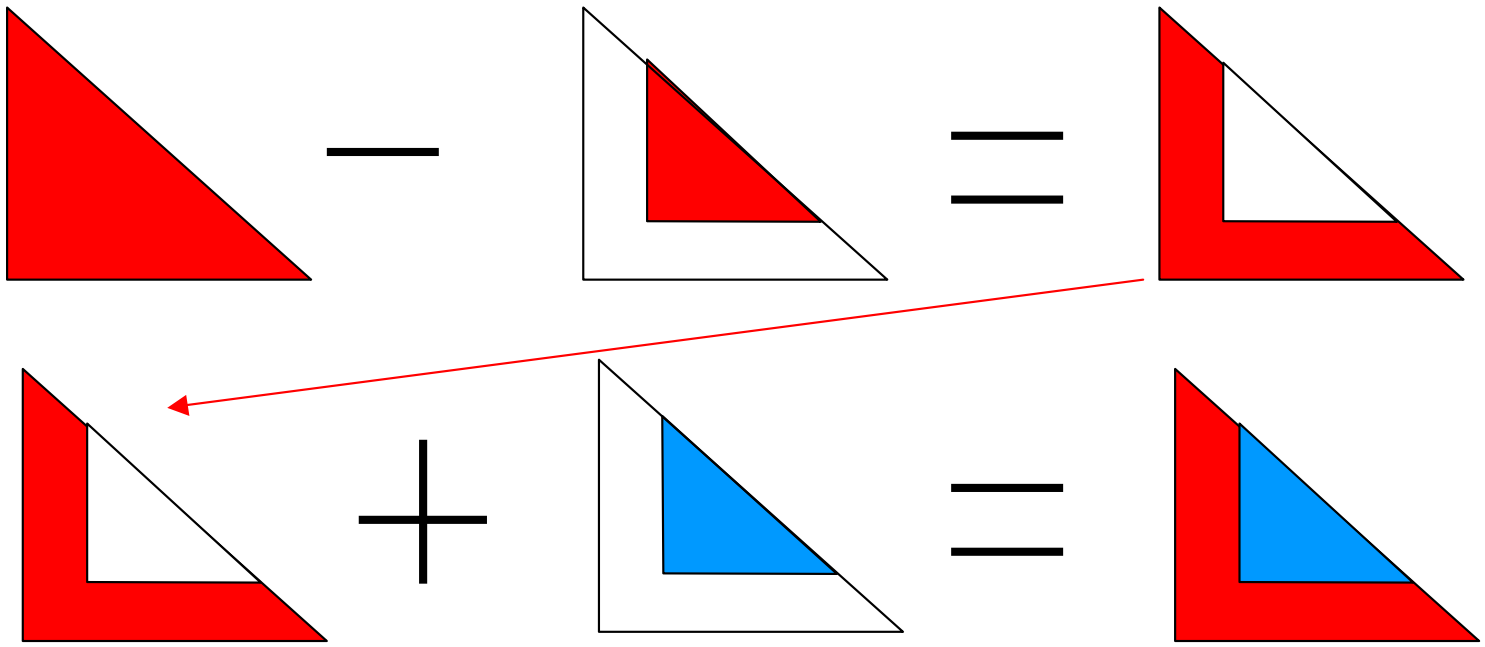
$$+ 2f_c\frac{\alpha_s}{2\pi}\int_0^1 dx\sigma_0(xs)\left[L\frac{1+x^2}{(1-x)_+} + 2\frac{(1+x^2)\ln(1-x)}{(1-x)_+} - \frac{1+x^2}{1-x}\ln x\right]$$

$$- 2f_c\frac{\alpha_s}{2\pi}\int_0^1 dx\sigma_0(xs)\frac{1+x^2}{(1-x)_+}\ln\left(\frac{s}{Q_c^2}(1-x)-1\right)\Theta\left(1-\frac{2Q_c^2}{s}-x\right),$$



$$L = \ln(s/\mu^2)$$

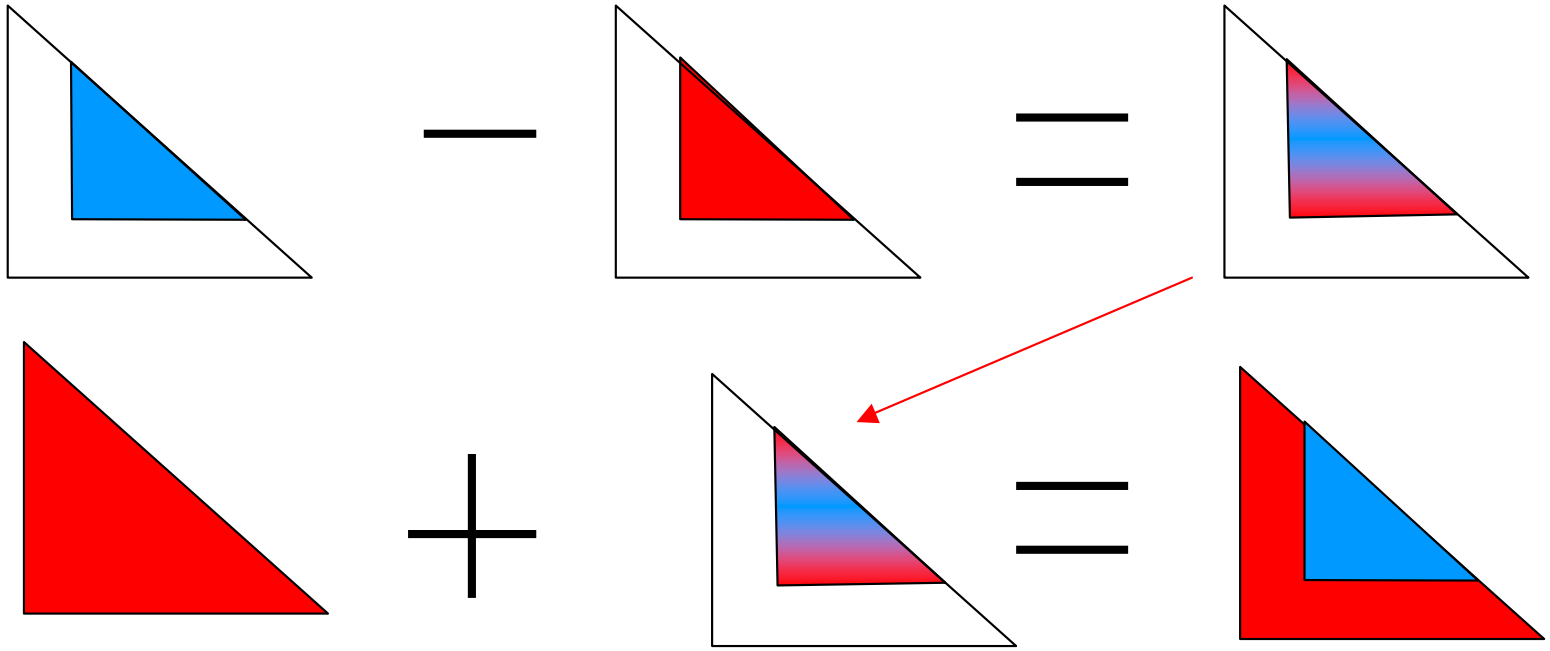
Phase space slicing



: Collinear (Leading Log) Approx.

: Exact Matrix Elements

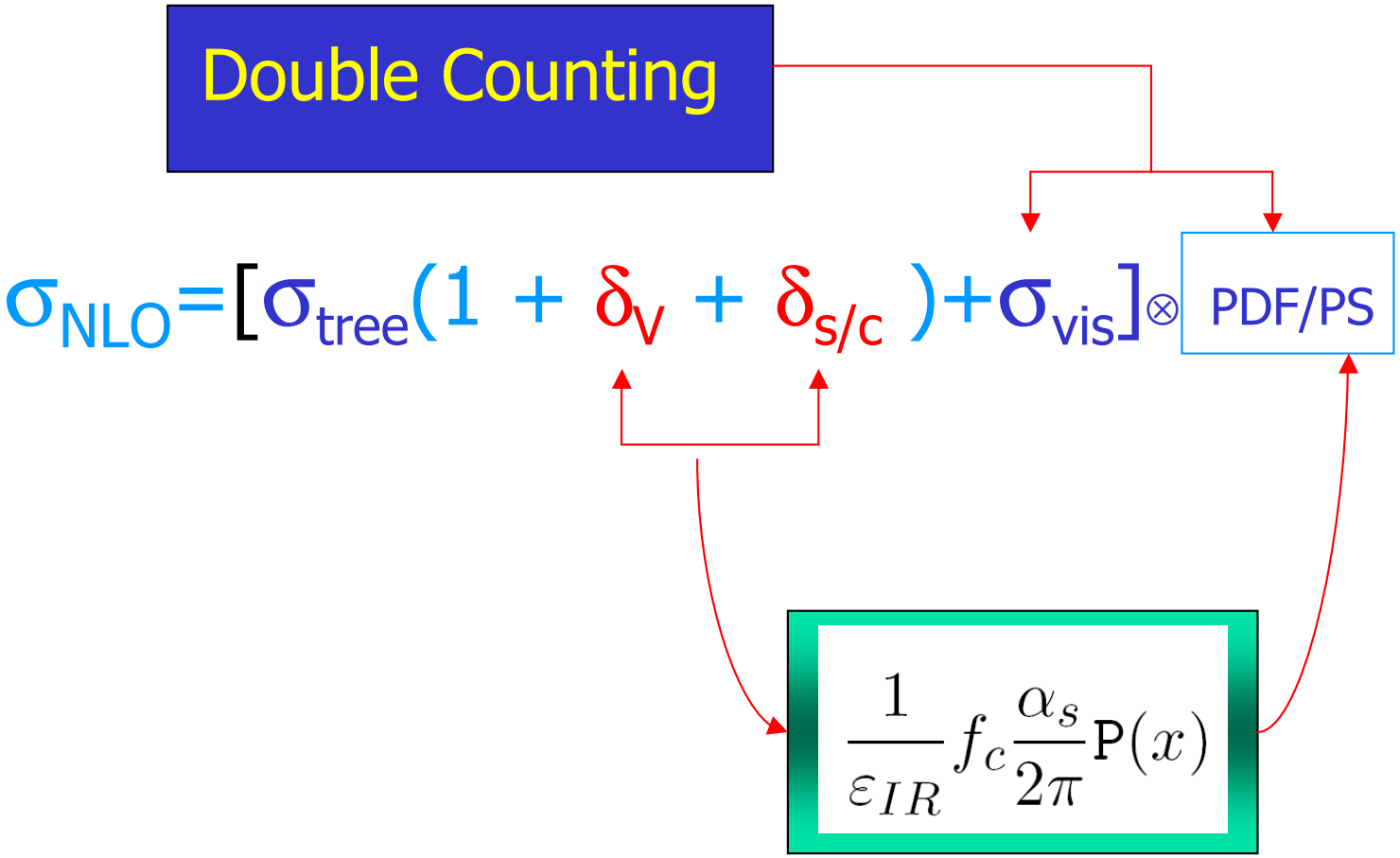
Leading Log Subtraction



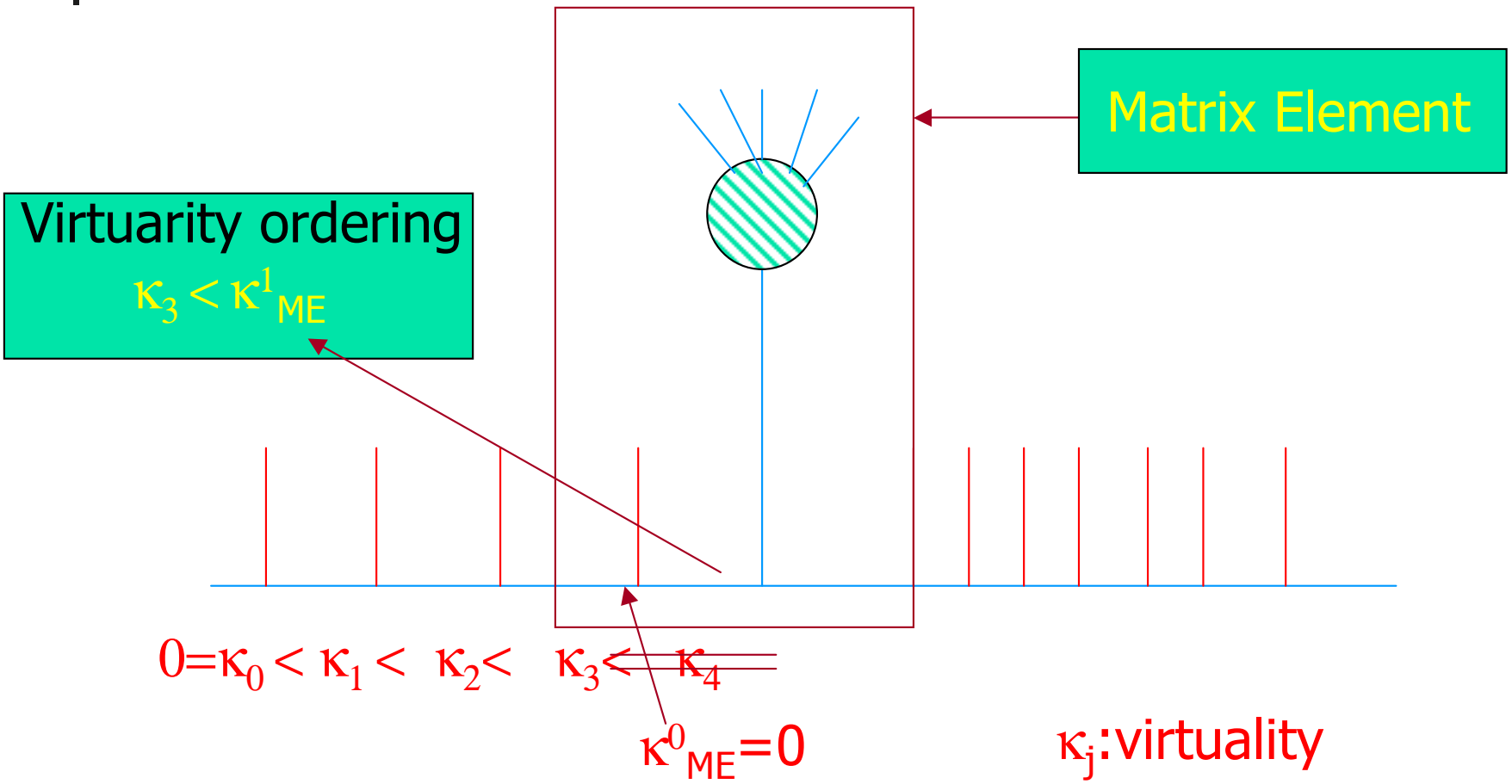
: Collinear (Leading Log) Approx.

: Exact Matrix Elements

Double Counting Problem

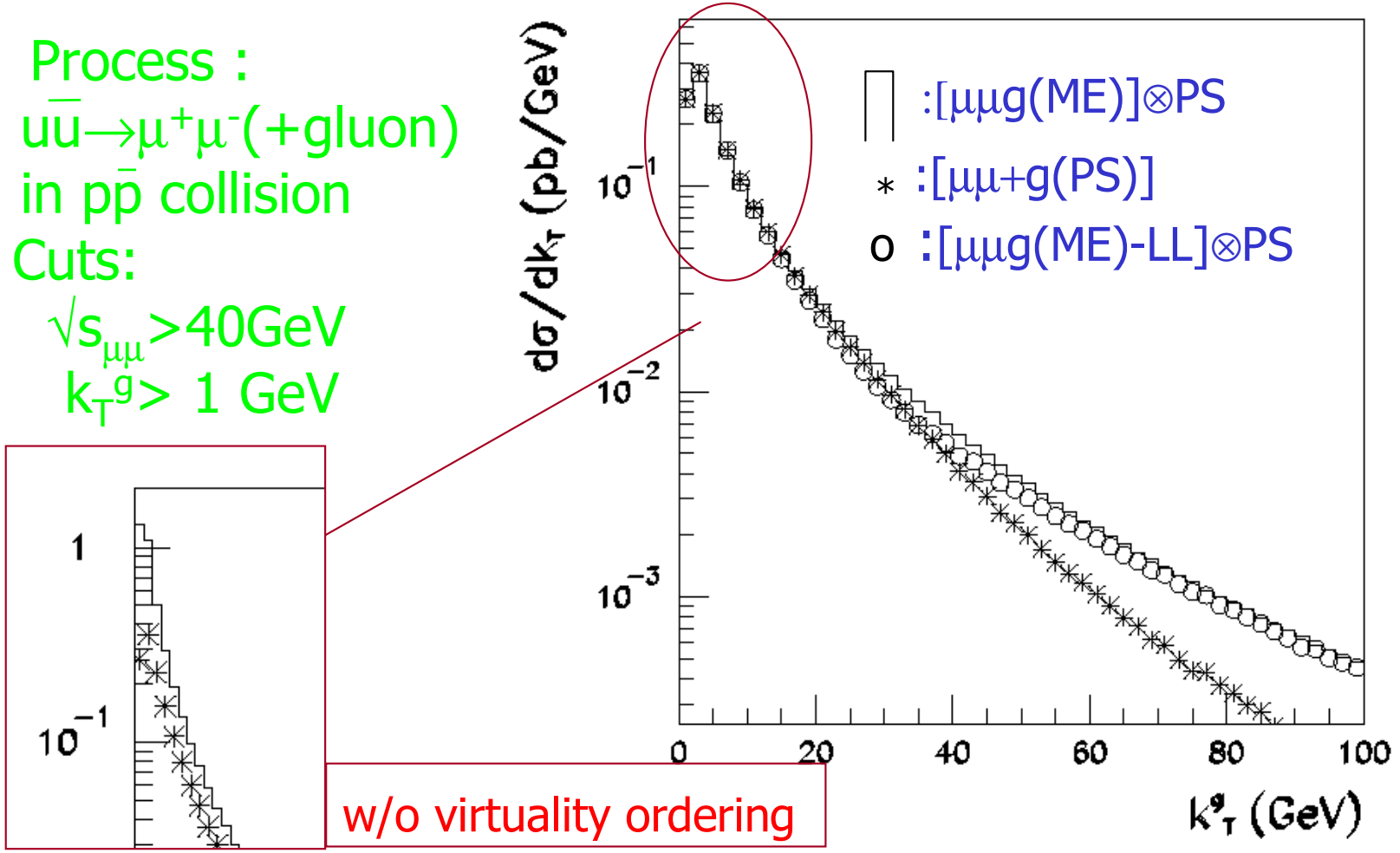


Double Counting Rejection

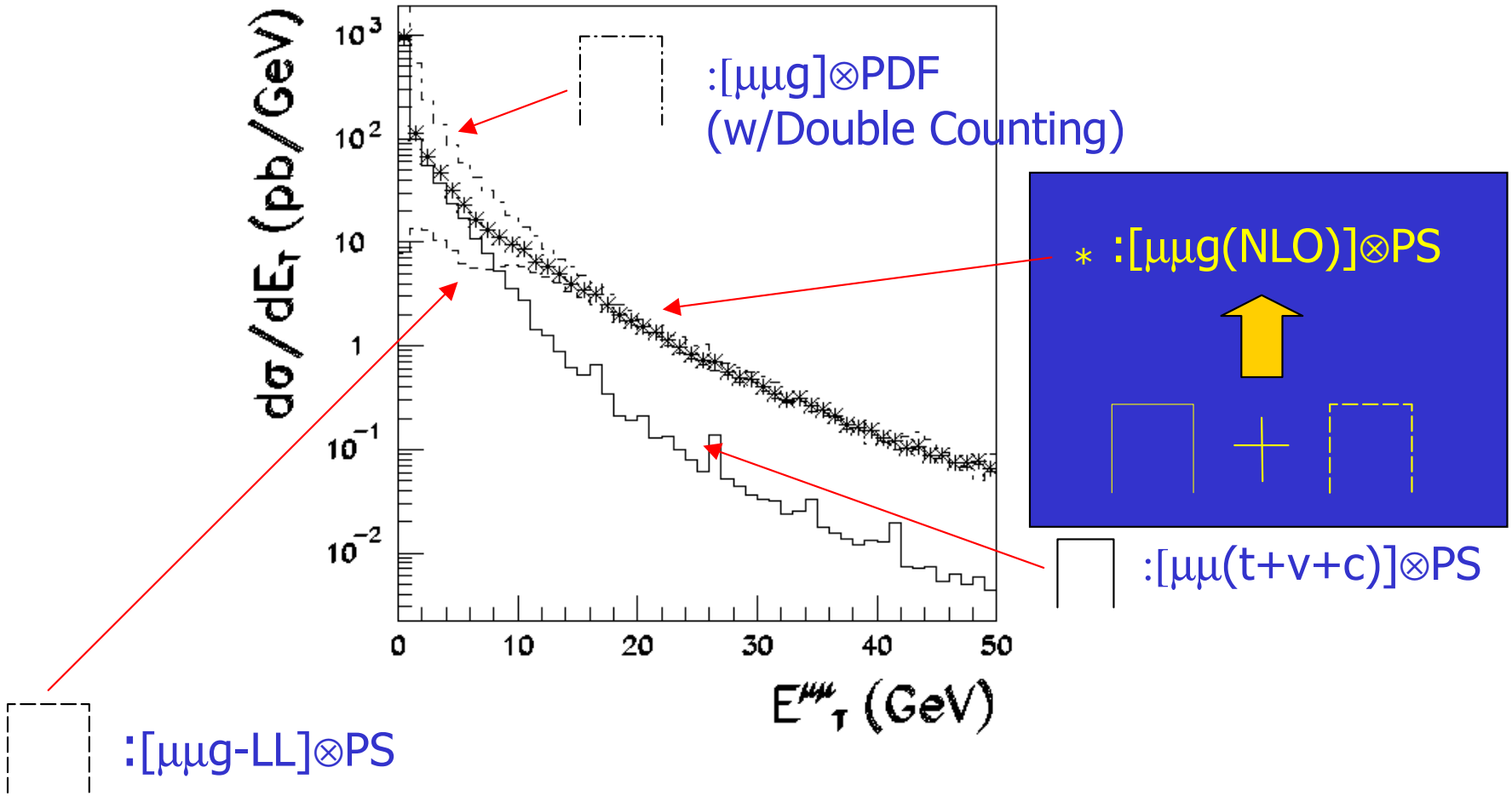


k^g_T Test:Drell-Yan Process

Process :
$u\bar{u} \rightarrow \mu^+\mu^-$(+gluon)
in $p\bar{p}$ collision
Cuts:
$\sqrt{s_{\mu\mu}} > 40$GeV
$k_T^g > 1$ GeV



E_T of μ-pair system



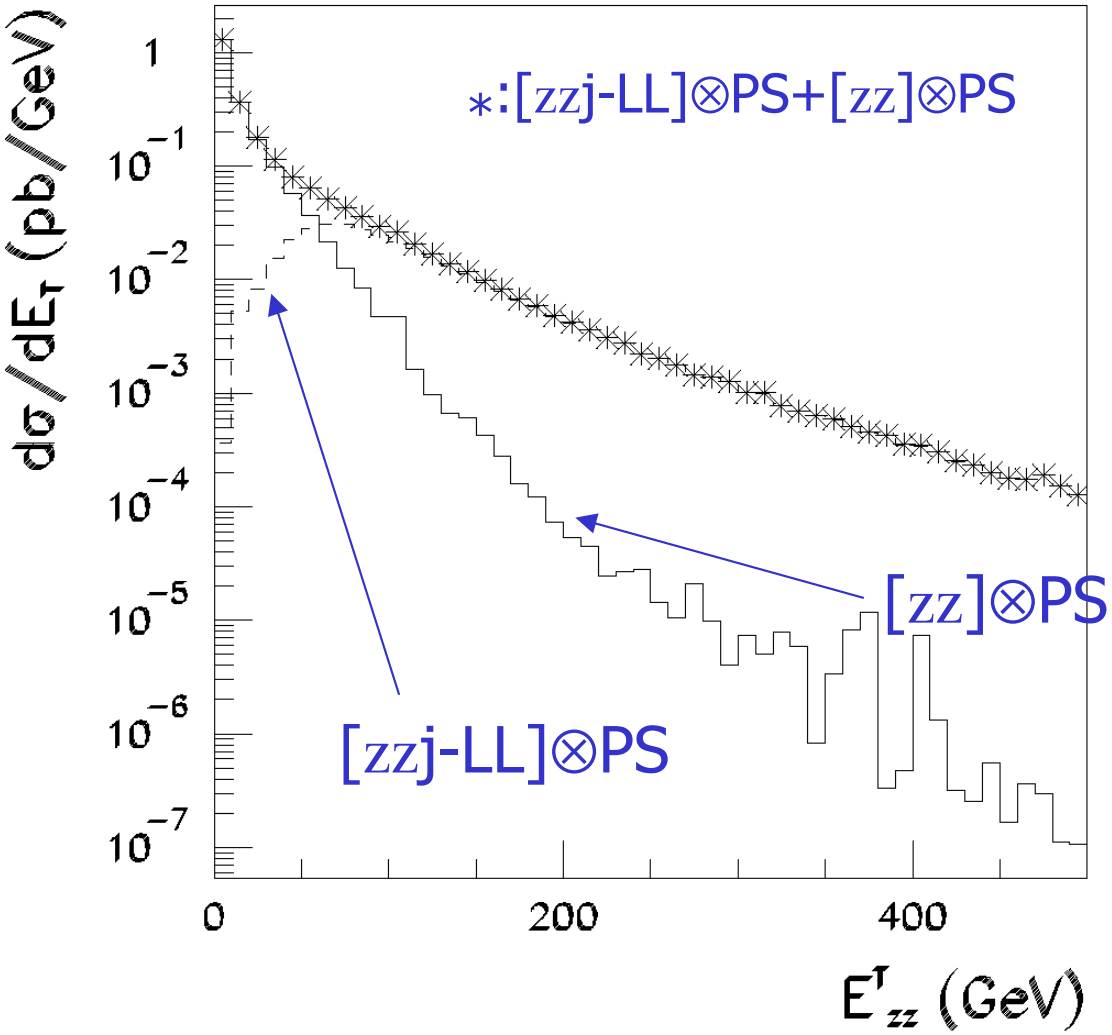
pp→zz+jet process

Process: pp→zz+jet

$\sqrt{s_{pp}}$=14 TeV

PDF : Cteq5L

(E_{scale}=Mz+$\overline{p_T}$)

Cuts: non

E_T of the zz-system