Final Results for the Solenoid Magnetic Field Map

CERN mapping project team

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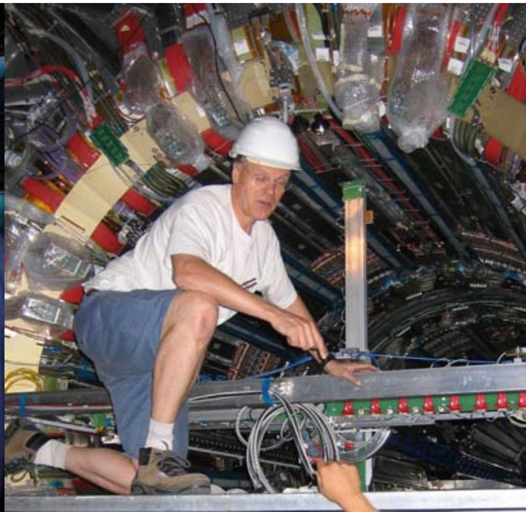
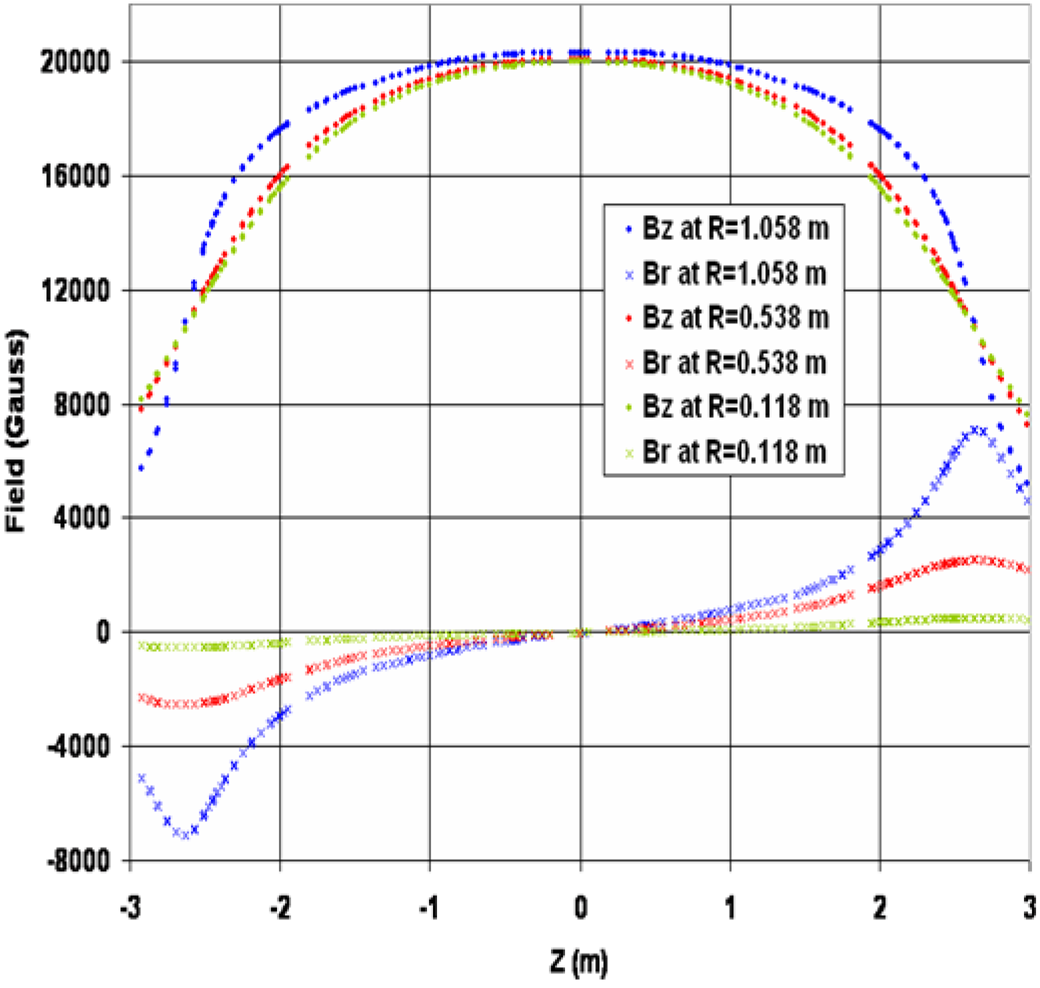


Outline

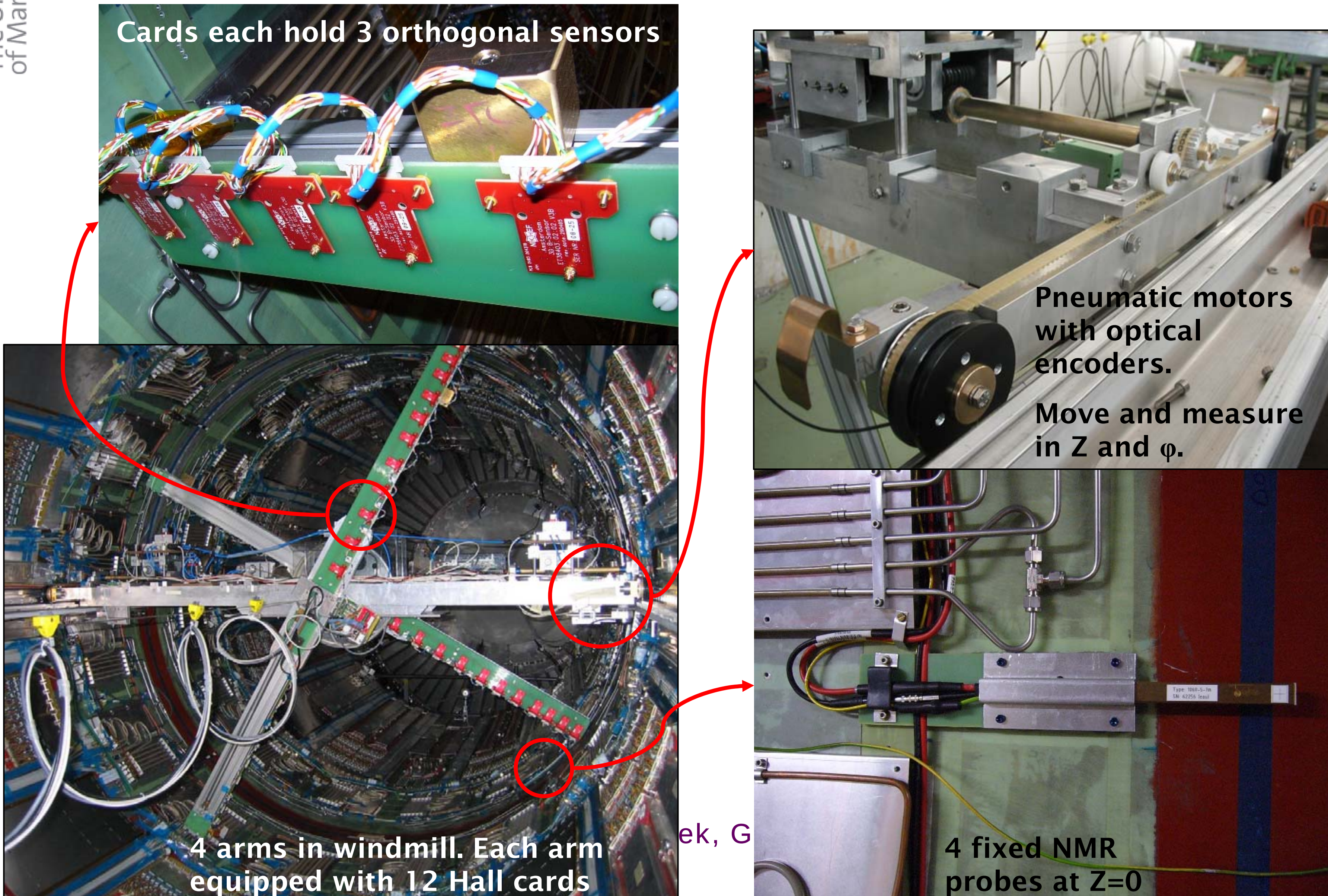
- Overview of mapping campaign
- Corrections to data
- Geometrical fit results
- Geometrical + Maxwell fit results
- Systematic errors
- Conclusions

Overview of the task

- Mapping 6m long x 2m diameter cylindrical volume
- 2 Tesla (20000 Gauss) at Z=0, dropping to 0.8 T at Z=3m
- Defines momentum scale of all ID tracks
- Require track sagitta error due to field uncertainty < 0.05%
  - Ensures that error in momentum due to field is less than error due to tracker alignment at p_T of 40 GeV



The field mapping machine



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Data sets recorded

- Data taken at four different solenoid currents
  - Nominal current (7730 A) gives 2 T at centre
  - Low current (5000 A) gives 1.3 T and is used with low-field probe calibration
- Fine phi scans used to measure the (tiny) perturbation of the field by the mapping machine
- Total data collected ~0.75M
  - Statistical errors will be negligible

Date in August	Current (A)	Number of φ steps	Number of Z steps
2nd-3rd	**7730**	**16**	**99**
3rd	**7730**	**64**	**1**
4th	**7850**	**16**	**25**
	7000	**16**	**44**
	5000	**16**	**76**
	5000	**64**	**1**
7th	**7730**	**16**	**8**
	7730	**24**	**26**
	7730	**12**	**35**

Corrections to data

Geometrical effects

- Plenty of survey data taken before and after mapping campaign
  - Positions of individual Hall sensors can be determined to ~0.2 mm accuracy
- Mapping machine skew recorded in data
- Carriage tilts determined from data

Probe calibrations

- Response of Hall sensors calibrated as function of field strength, field orientation and temperature using test stands at CERN and Grenoble
  - Low-field calibration (up to 1.4 T) has expected accuracy of ~2 G, 2 mrad
  - High-field calibration (up to 2.5 T) has expected accuracy of ~10 G, 2 mrad
- NMR probes intrinsically accurate to 0.1 G
- Absolute scale of high-field Hall calibration improved using low-field Hall calibration and NMR values
- Relative Hall probe normalisations and alignments determined from data

Other effects

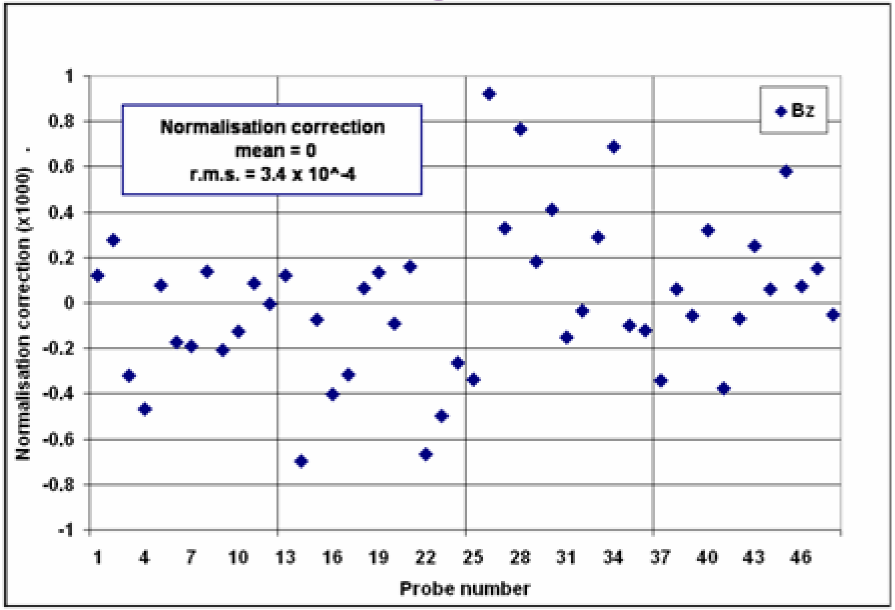
- Effects of magnetic components of mapping machine corrected using magnetic dipoles

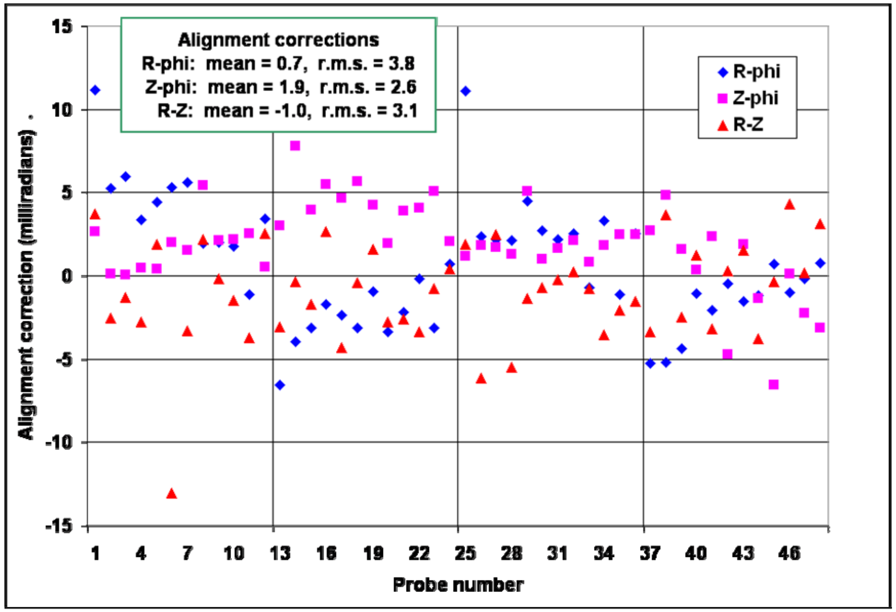
Probe normalisation and alignment

- Exploited the mathematics of Maxwell's equations to determine relative probe normalisations and alignments
- B_Z normalisation:
  - Uses the fact that each probe scans the field on the surface of a cylinder
  - B_Z at centre determined for each probe
  - All probes were then normalised to the average of these values
- Probe alignment:
  - Uses curl **B** = **0** and

$$B_\varphi^m = B_\varphi + A_{\varphi z} B_z - A_{r\varphi} B_r$$

  - Integrate $\oint B_\varphi ds = 0$
  - Tilt angles A_{ij} of probe were determined from a least squares fit
  - The third alignment angle comes from div B = 0



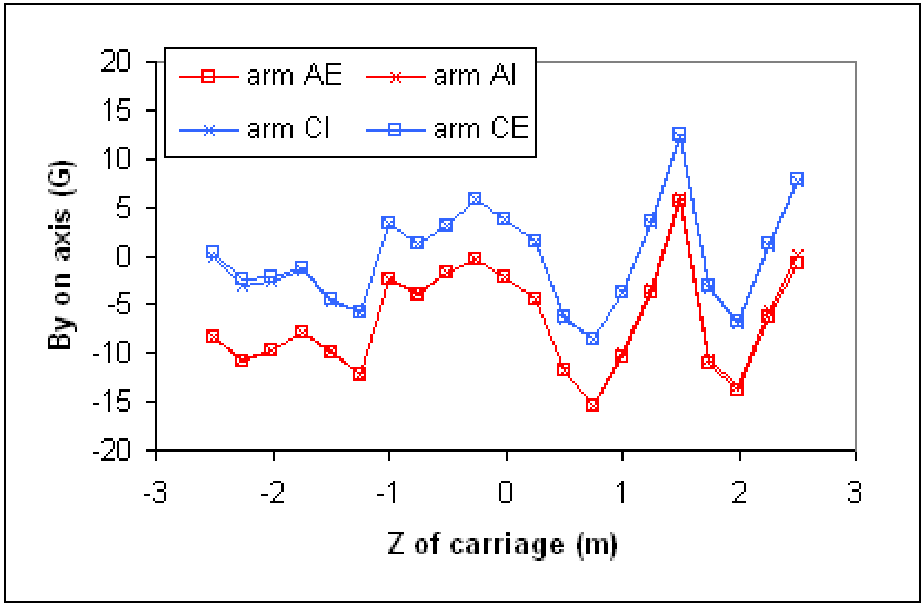


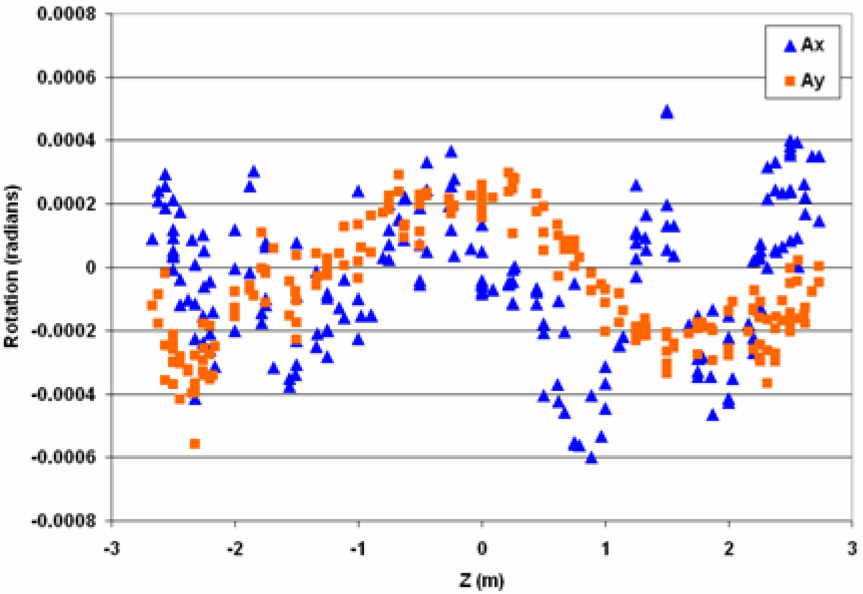
Carriage tilts

- Another analysis which exploited mathematics of Maxwell
- B_x and B_y on the z-axis evaluated from average over φ for probes near centre of solenoid
- Plots of B_x,B_y versus Z of carriage show evidence that entire carriage is tilting
- Degree of tilt can be calculated by integrating to find expected B_x,B_y value

$$\frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x}$$

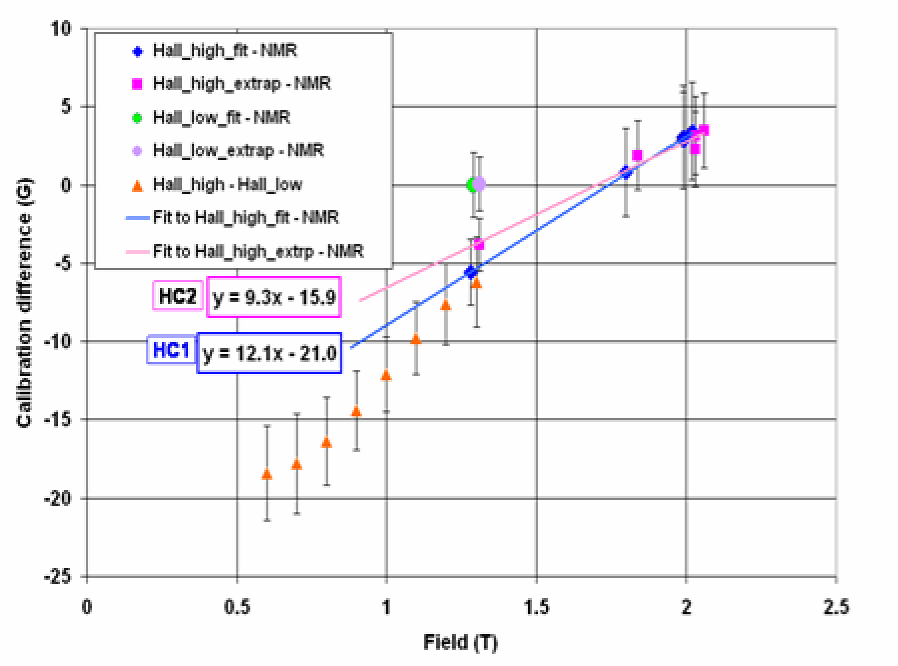
- Jagged structure of tilts suggest that machine is going over bumps on the rail of ~0.1 mm





Absolute Hall scale

- Absolute scale of high-field Hall calibration (10 G) is greatest uncertainty
  - Can be improved using low-field Hall calibration (2 G) and NMR value (0.1 G)
- Low-field Hall values and NMR values are equal for 5000 A data
  - Low-field Hall values are considered accurate in low-field region
- Discrepancy between low- and high-field Hall values in low-field region



- Discrepancy between high-field Hall values (derived from field fits) and NMR value in high-field region
  - This discrepancy lines up with the discrepancy from low-field region
- Alternative high-field Hall values from extrapolation give estimate of systematic error

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Fit quality measures

- We fit the map data to field models which obey Maxwell's equations
  - The volume covered has no currents and has effects of magnetic materials removed
  - Maxwell's equations become

$$\nabla \cdot B = 0 ; \quad \underline{\nabla} \times \underline{B} = \underline{0}$$

- Our fit uses Minuit to minimise

$$\chi^2 = \sum_{\substack{i=\text{data points} \\ c=Z,R,\phi}} \left(\frac{B_{c,i}^{\text{measured}} - B_{c,i}^{\text{fit}}}{5\,\text{Gauss}} \right)^2$$

- Our aim is to know the track sagitta, which is proportional to (c_r and c_z are direction cosines)

$$S = \int_0^{r_{\max}} r\left(r_{\max} - r\right)\left(c_r B_z - c_z B_r\right) dl$$

- Our fit quality is defined to be $\delta S/S$ where

$$\delta S = \int_0^{r_{\max}} r\left(r_{\max} - r\right)\left(c_r\left(B_z^{\text{meas}} - B_z^{\text{fit}}\right) - c_z\left(B_r^{\text{meas}} - B_r^{\text{fit}}\right)\right) dl$$

Geometrical fit

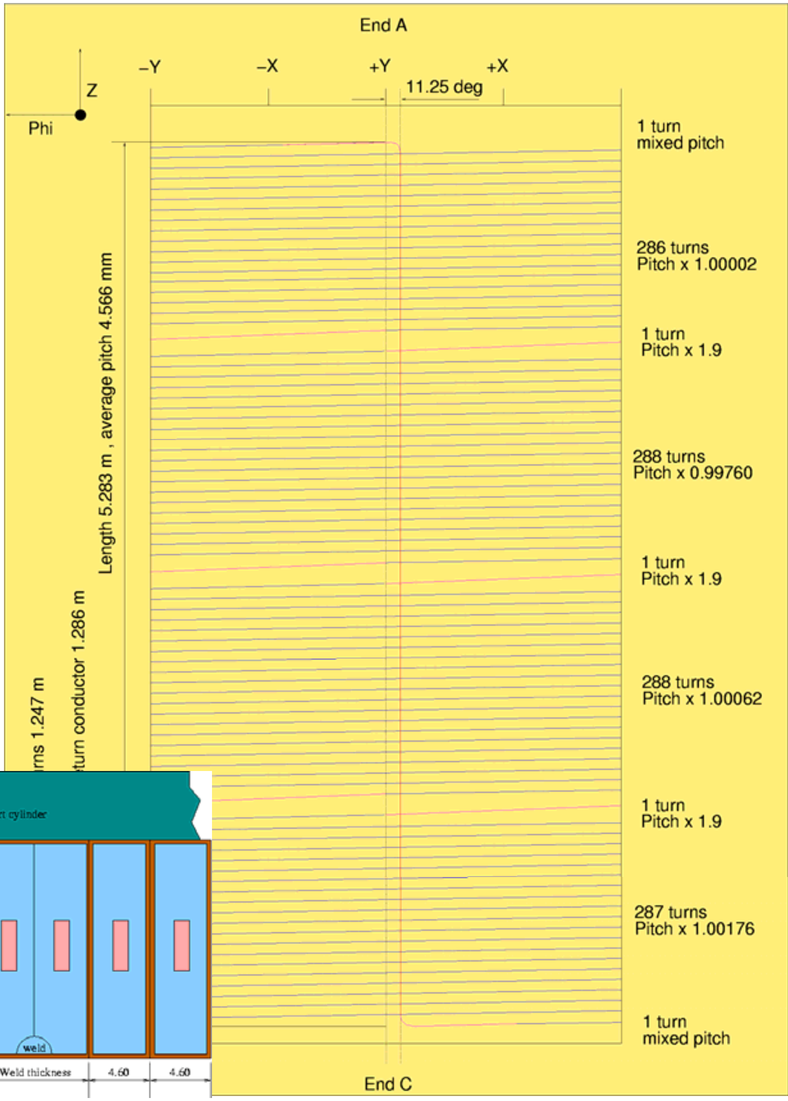
- 96% of the field is directly due to the solenoid current
  - We use a detailed model of the conductor geometry and integrate Biot-Savart law using the known current
  - 7 free parameters
    - Scale factor and aspect ratio (length/diameter) of conductor model
    - Position and orientation of conductor model relative to IWV
- 4% of field is due to magnetised iron (TileCal, girders, shielding discs etc)
  - Parametrised using 4 free parameters of Fourier-Bessel series with length scale=2.5m

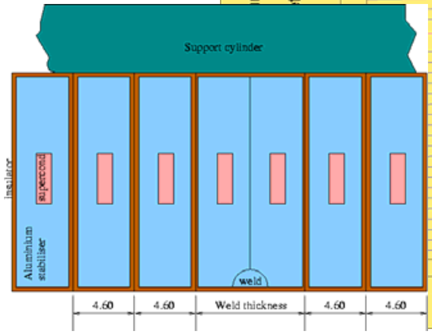
$$B_z = \sum_{i=1}^{4} C_i \cos\left(\frac{zi}{s}\right) I_0\left(\frac{ri}{s}\right)$$

$$B_r = \sum_{i=1}^{4} C_i \sin\left(\frac{zi}{s}\right) I_1\left(\frac{ri}{s}\right)$$

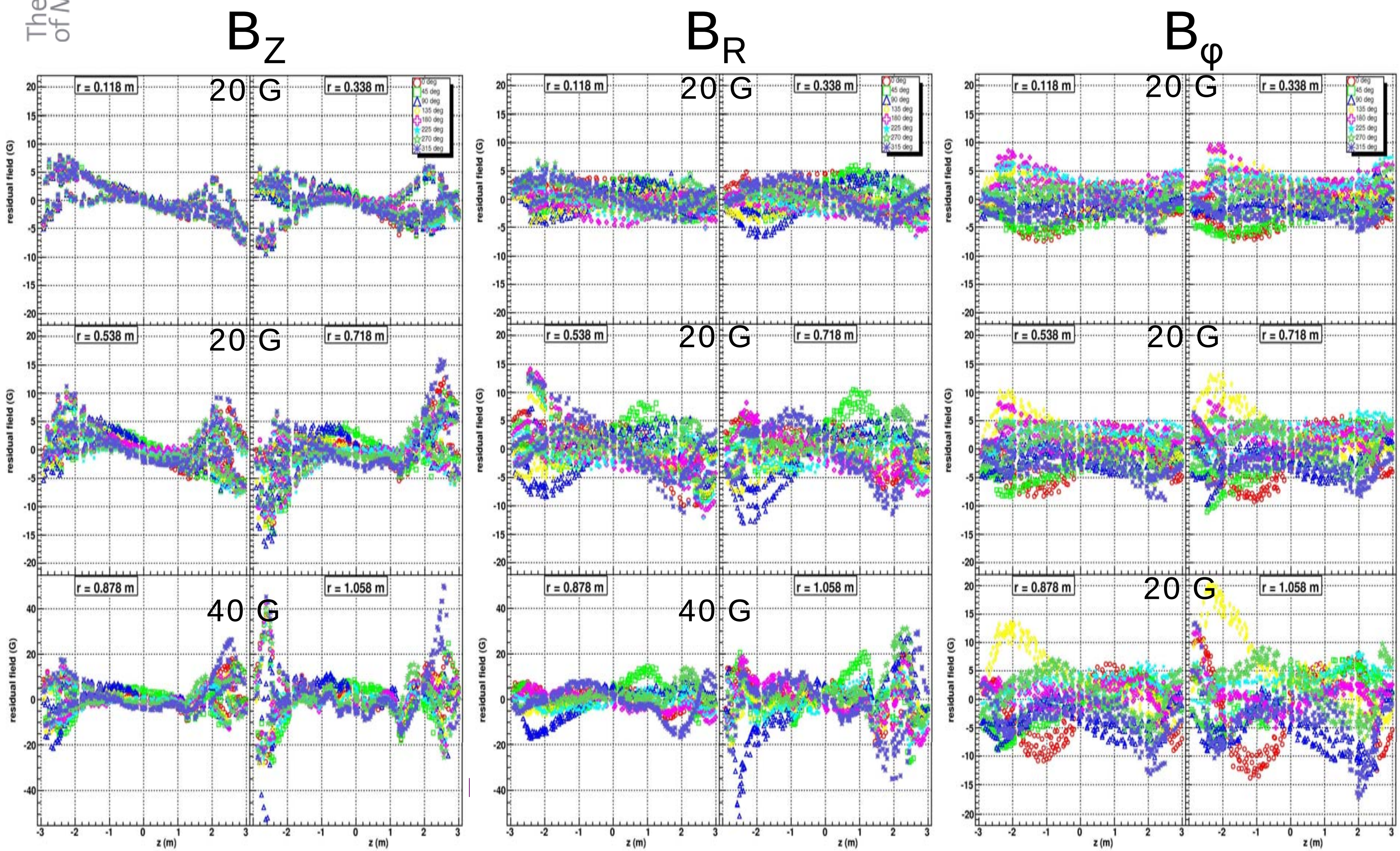
Conductor geometry determined by surveys of solenoid as built

except weld thickness, which was determined from data as 1.9×pitch





Results from geometrical fit I



Results from geometrical fit II

- Residuals + sagitta error calculated for all data samples
  - 7730a used for our final fit results at nominal 2 T field
- Sagitta error rms is within our 5×10^{-4} target for all samples

Map	B_Z (G)		B_R (G)		B_φ (G)		$\delta S/S$ ($\times 10^{-4}$)	
	rms	extreme	rms	extreme	rms	extreme	rms	extreme
5000	2.96	-32.6	2.91	-41.4	2.80	-13.5	3.11	+12.0
5000h	4.12	-39.6	3.76	-43.2	3.23	-14.8	3.63	+13.5
7000	5.82	+52.9	5.41	-48.7	4.66	+21.6	3.14	+10.6
7730a	5.23	-51.5	5.14	-49.9	4.60	+22.1	3.35	+10.9
7730b	4.45	+50.2	4.81	-48.9	4.57	-24.6	3.20	-11.6
7850	4.59	+47.9	5.02	-48.8	4.90	-22.4	2.92	+10.4

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Results from geometrical fit III

- Fit parameters tabulated for all samples
- All parameters consistent with expected results
  - Expected offsets for solenoid centre: x = -0.3 ± 0.4 mm, y = -2.2 ± 0.4 mm, z = -0.1 ± 2.3 mm
  - Z and R scale factors expected to be 1

Map	Offsets (mm)			Angles (mrad)		Scale factors		% iron at orig
	x	y	z	A_x	A_y	Z	R	
5000	0.44	-2.52	0.36	0.11	0.09	1.00158	0.99900	4.108
5000h	0.47	-2.46	0.35	0.11	0.09	1.0015	0.9991	4.101
7000	0.38	-2.36	0.49	0.15	0.07	1.0014	0.9992	4.075
7730a	0.28	-2.39	0.51	0.13	0.09	1.00121	0.99926	4.0512
7730b	0.29	-2.38	0.56	0.10	0.12	1.00119	0.99932	
7850	0.34	-2.51	0.60	0.12	0.12	1.0013	0.9995	4.060

Full fit (geometrical + Maxwell)

- A few features remain in the residuals from the geometrical fit
  - Ripples for |Z|<2m believed to be due to variations in the coil winding density
  - Bigger features at |Z|>2m believed to arise from the coil cross-section not being perfectly circular
    - These effects are more pronounced at the ends of the solenoid
- These features cannot be determined accurately enough to be included in the geometrical model
  - However, they are real fields which should obey Maxwell's equations
- We apply the general Maxwell fit to the residuals to account for these features

General Maxwell fit

- General fit able to describe any field obeying Maxwell's equations.
- Uses only the field measurements on the surface of a bounding cylinder, including the ends.
- Parameterisation proceeds in three stages:

  1. B_z on the cylindrical surface is fitted as Fourier series, giving terms with φ variation of form $\cos(n\varphi+\alpha)$, with radial variation $I_n(\kappa r)$ (modified Bessel function).
  2. $B_z^{\text{meas}} - B_z^{(1)}$ on the cylinder ends is fitted as a series of Bessel functions, $J_n(\lambda_j r)$ where the λ_j are chosen so the terms vanish for $r = r_{\text{cyl}}$. The z-dependence is of form $\cosh(\mu z)$ or $\sinh(\mu z)$.
  3. The multipole terms are calculated from the measurements of B_r on the cylindrical surface, averaged over z, after subtraction of the contribution to B_r from the terms above. (The only relevant terms in B_z are those that are odd in z.)

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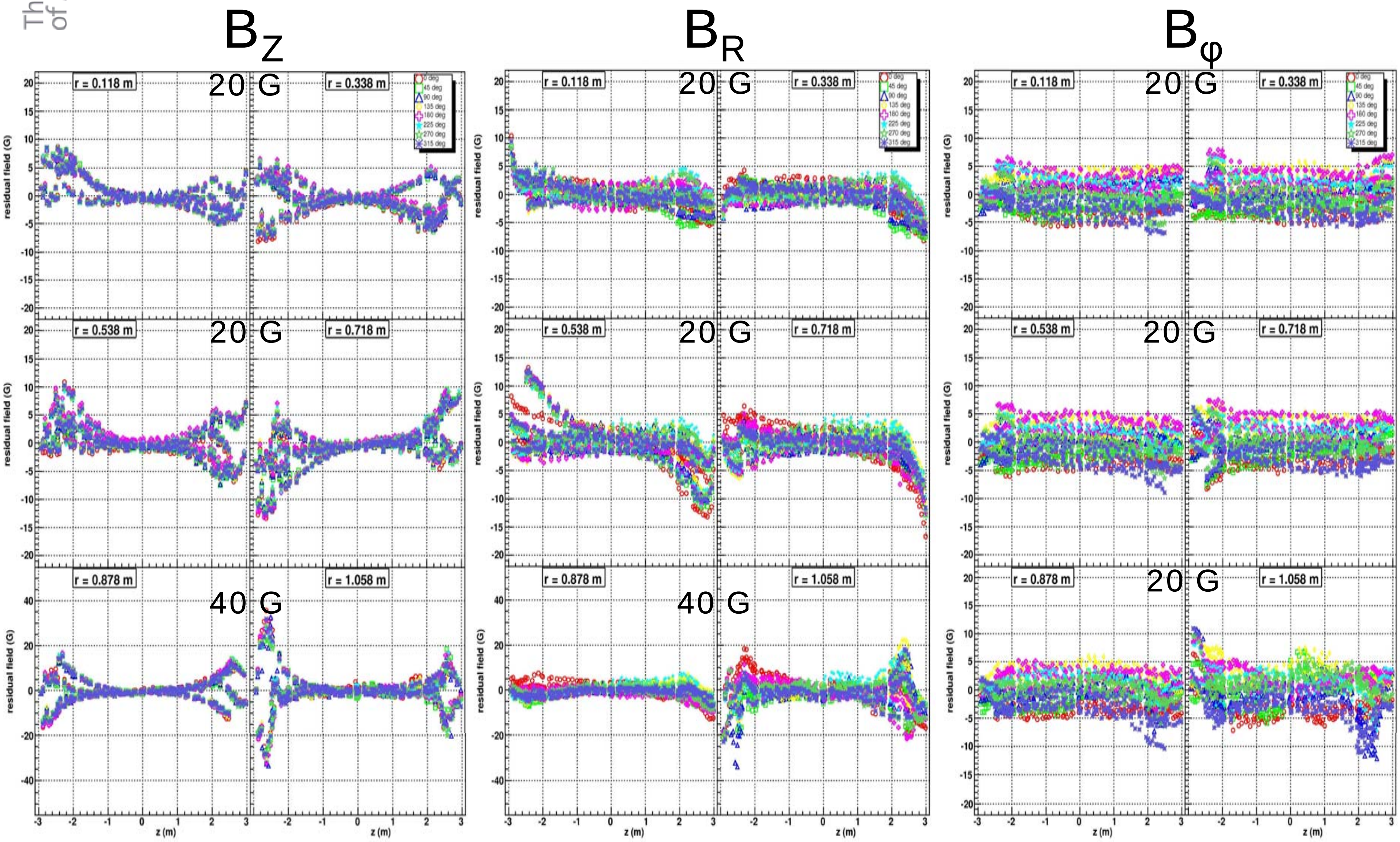
Results from full fit I



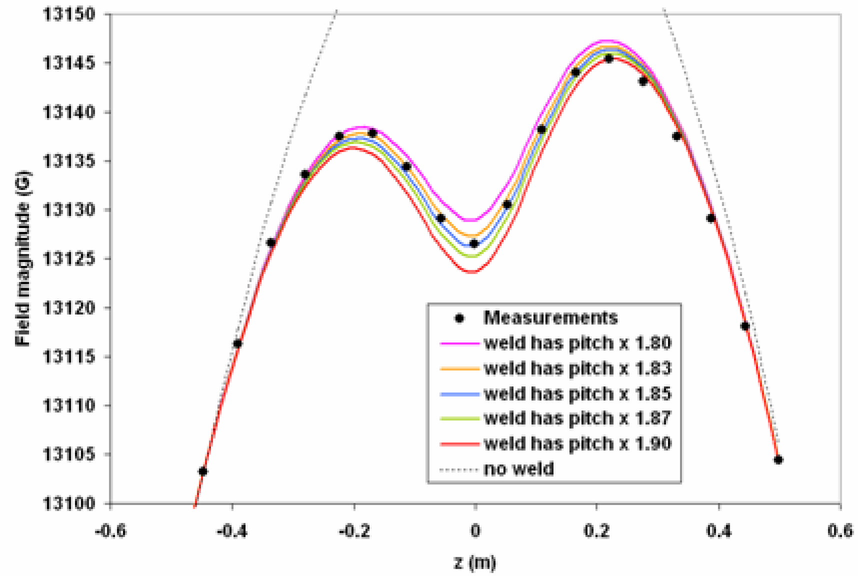
Results from full fit II

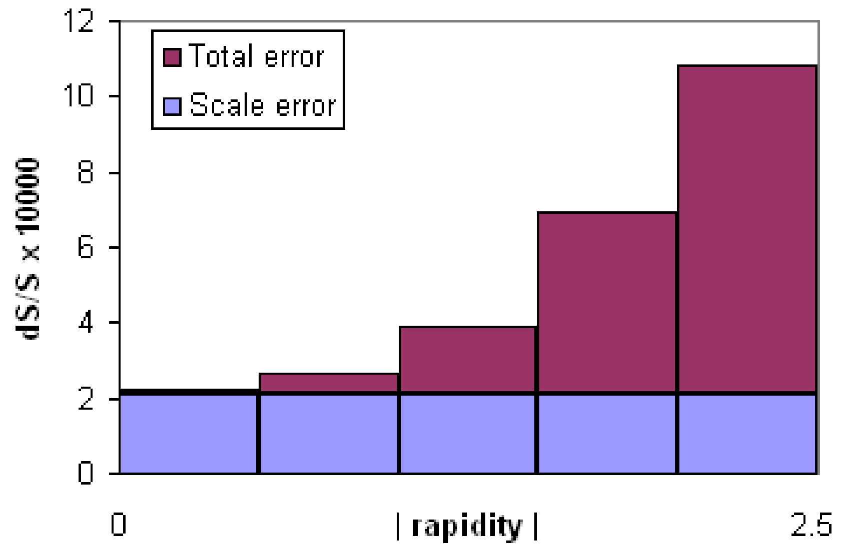
- Residuals of all probes reduced significantly
  - Recall that Maxwell fit is made using outermost probes only
  - Fact that the fit matches inner probes as well shows strong evidence that the difference between data and geometrical model is a real field
- Fit quality $\delta S/S$ also improved at high η

Map	B_Z (G)		B_R (G)		B_φ (G)		$\delta S/S$ ($\times 10^{-4}$)	
	rms	extreme	rms	extreme	rms	extreme	rms	extreme
5000	2.27	-25.1	1.84	-30.1	1.85	+11.5	1.70	+6.2
5000h	3.68	-31.0	3.12	-28.3	2.75	+12.7	2.40	+9.9
7000	4.97	-37.5	4.49	-33.5	3.64	+15.9	1.51	+7.2
7730a	4.34	-37.1	3.52	-33.8	2.90	+15.2	1.29	+6.5
7730b	3.47	-32.3	3.74	-54.1	3.85	+17.0	1.58	+8.4
7850	3.55	-32.6	3.85	-48.8	3.85	-17.2	1.69	+9.0

Systematic errors

- Uncertainty in overall scale
  - Spread in Hall-NMR differences over 4 NMR probes
  - Weld thickness, which influences the Hall-NMR comparison
  - Overall scale error 2.1×10^{-4}
- Uncertainty in shape of field
  - Considered several "reasonable" changes to fit model
  - Dominant factor is 0.2 mrad uncertainty in orientation of the rotation axes of the mapping machine arms relative to IWV coordinates
  - Overall shape error 5.9×10^{-4}
- Total uncertainty 6.3×10^{-4}
  - Dominated by scale error at low η, shape error at high η





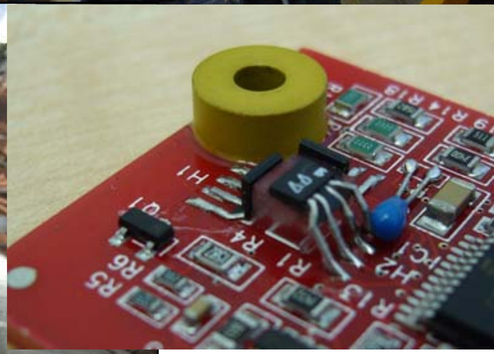
Conclusions

- The solenoid field mapping team recorded lots of high quality data during a successful field mapping campaign
- All possible corrections from surveys, probe calibrations and probe alignments have been applied to the data
- We have determined a fit function satisfying Maxwell's equations which matches each component of the data to within 4 Gauss rms
- Using this fit, the relative sagitta error is 6.3×10^{-4}
- At high rapidity, the systematic errors are dominated by a 0.2 mrad uncertainty in the direction of the field axis relative to the IWV coordinate system

Backup slides

Surveys

- Survey of mapping machine in Building 164
  - Radial positions of Hall cards
  - Z separation between arms
  - Z thickness of arms
- Survey in situ before and after mapping
  - Rotation centre and axis of each arm
  - Position of Z encoder zero
  - Positions of NMR probes
- Survey of ID rails
  - Gradient wrt Inner Warm Vessel coordinates
- Survey of a sample of 9 Hall cards
  - Offsets of BZ, BR, Bf sensors from nominal survey point on card
- Sensor positions known with typical accuracy of 0.2 mm



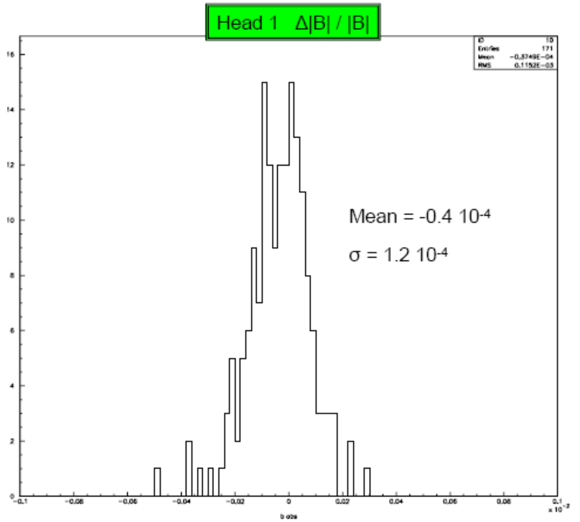
Probe calibrations

Hall sensors

- Response measured at several field strengths, temperatures and orientations (θ,φ)
- Hall voltage decomposed as spherical harmonics for (θ,φ) and Chebyshev polynomials for |B|,T

$$V(|B|, Temp, \theta, \varphi) = \sum_{k}\sum_{n}\sum_{l}\sum_{m=0}^{l} c_{klm} T_k(|B|) d_{nlm} T_n(Temp) Y_{lm}(\theta, \varphi)$$

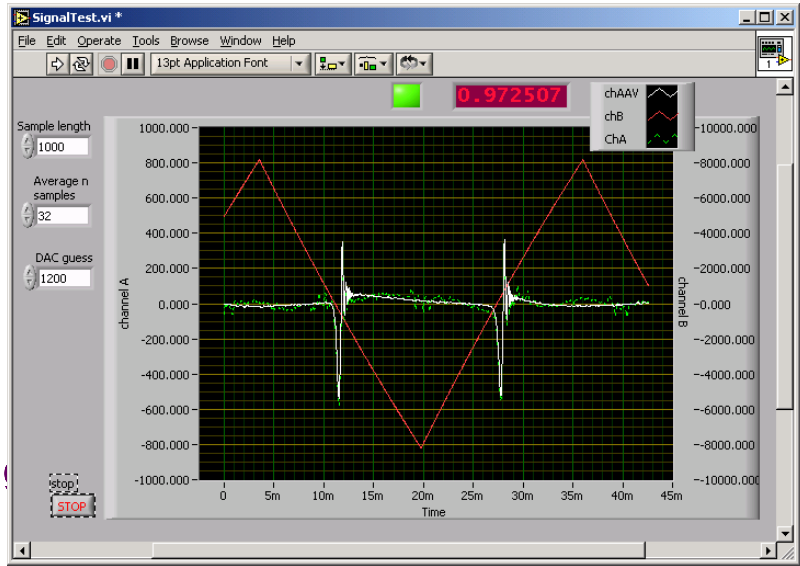
- **Low-field calibration (up to 1.4 T):** expected accuracy ~2 G, 2 mrad
- **High-field calibration (up to 2.5 T):** expected accuracy ~10 G, 2 mrad



B measured by NMR compared to B from sensors at 0.885 Tesla

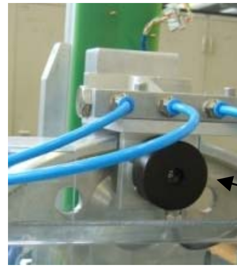
NMR probes

- No additional calibration needed (done by whoever measured Gp = 42.57608 MHz/T)
- Compare proton resonance frequency with reference oscillator
- **Intrinsically accurate to 0.1 G**

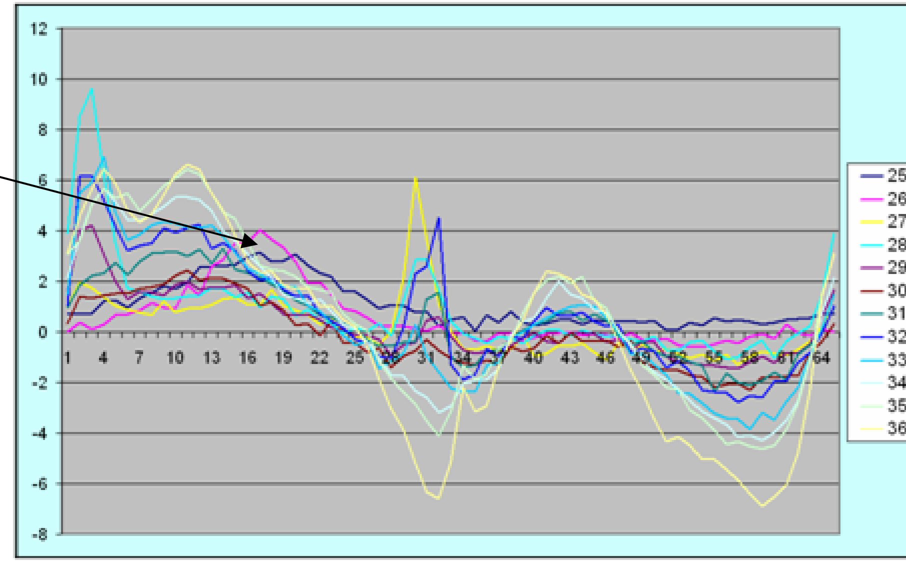


Magnetic machine components

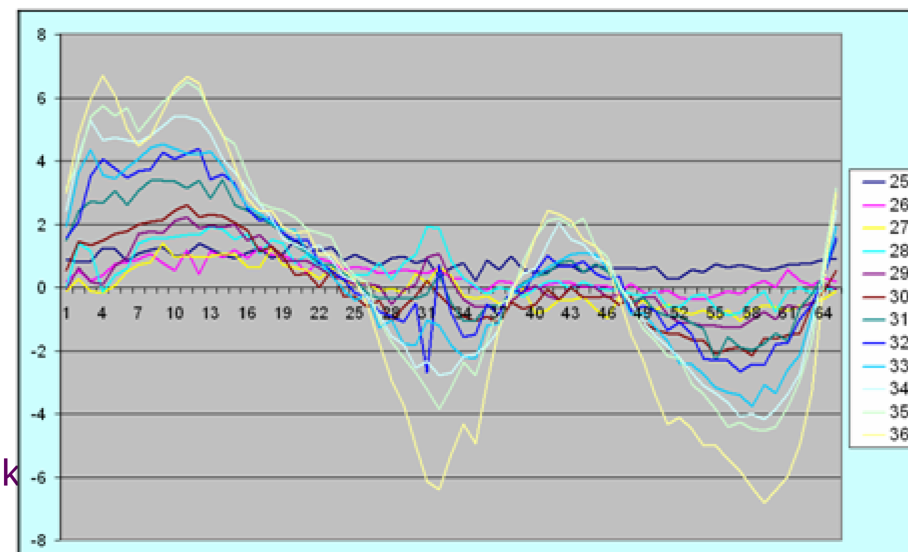
- Perturbation of the magnetic field by the mapping machine was not anticipated



- Some spikes in the data were clearly attributed to components of the mapping machine
- A dipole was subtracted at each component position with field strength chosen to make residuals look smooth

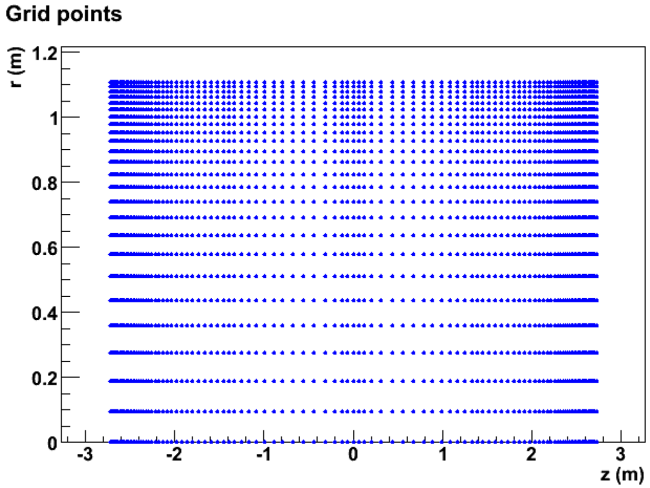


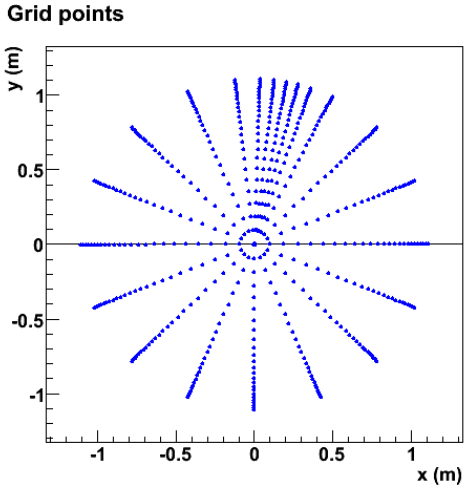
Object	Z (m)	R (m)	Phi (deg)	Strength
Phi encoder	-0.02	0.190	90	0.0090
Phi motor bearings	-0.13	0.378	164	0.0023
Z motor bearings	-0.13	0.772	171	0.0023
Magnetic plug on ESB	-0.04	0.457	9	0.0182
Z encoder	-0.04	1.080	188	0.0056
Z encoder	0.00	1.080	352	0.0056
electrical valve	-0.08	0.865	18	0.0032
electrical valve	-0.08	0.830	162	0.0032
Z motor bearings	-0.13	0.830	8	0.0023



Implementation in athena I

- Map data to be stored in athena as a table of field map values on a grid
  - Cylindrical grid with 113×25×21=59325 points
  - O(2.5 MB) of memory
  - Linear interpolation between grid points





Extra grid points near special features (ends of solenoid, welds, return conductor)

Implementation in athena II

- Linear interpolation in each direction
- Residuals <3 G throughout most of ID
- Still need to optimise map values